Introduction to NTMs and Role of Rotation

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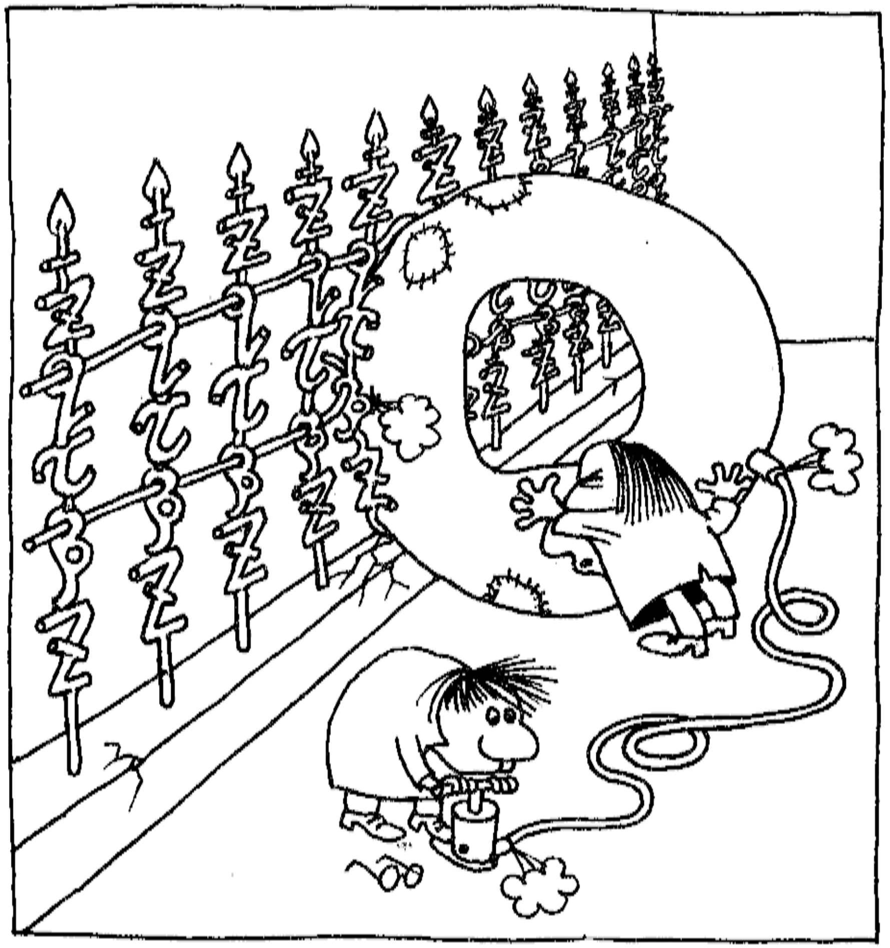
Tokamak Instabilities



(courtesy T. Hender)

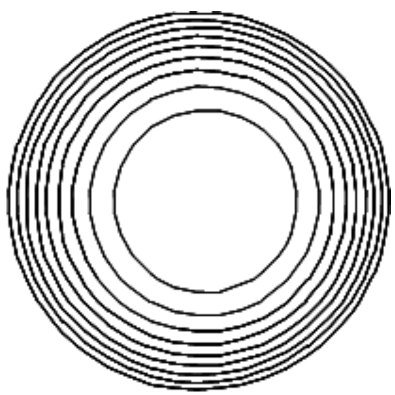
- Tokamaks are not minimum energy systems
- They contain pressure and current which can drive instabilities

Tokamak Instabilities

Magnetic field perturbation: $\vec{b} = \nabla\varphi \times \nabla\tilde{\psi}$

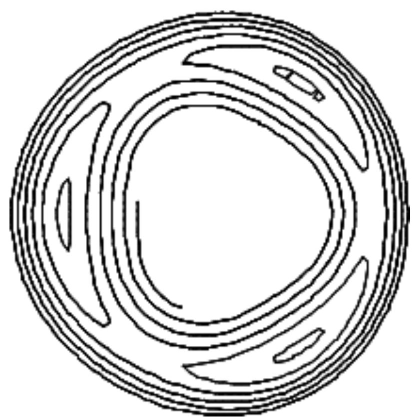
$$\tilde{\psi}(\varphi,\vartheta,r) = \tilde{\psi}_0(r)\cdot e^{\,i(n\varphi - m\vartheta)}$$

$q(r) = \frac{m}{n}$ – inside the plasma $\quad$ $q(r) = \frac{m}{n}$ – outside the plasma



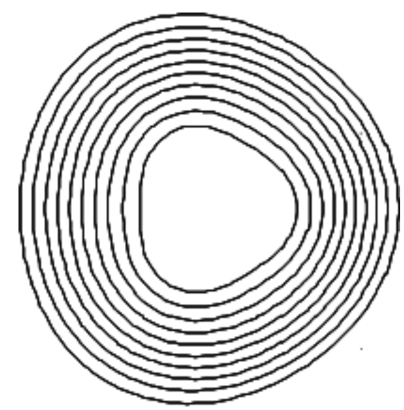
Unperturbed Magnetic Surfaces

▶



Internal (Tearing) Modes

or



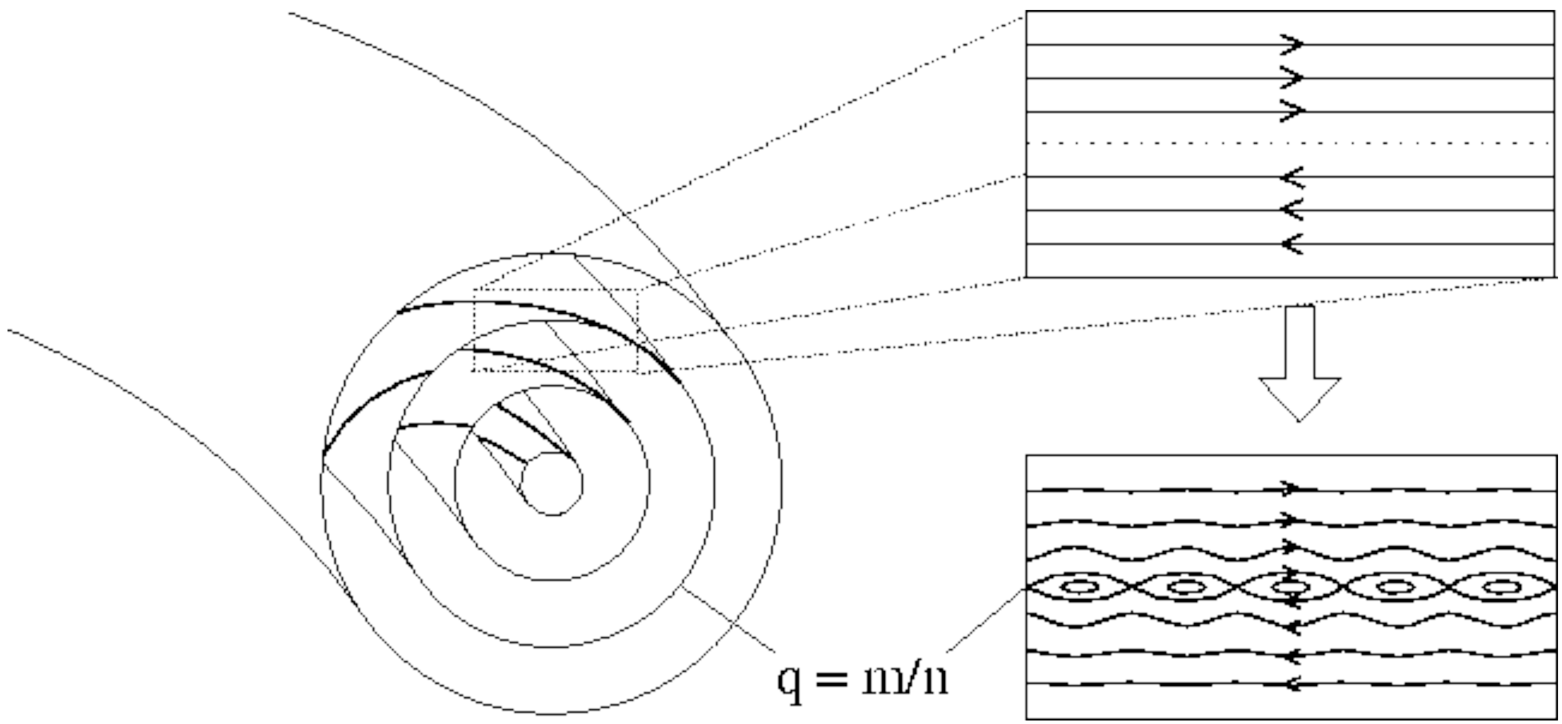
External Kink Modes

OUTLINE

- What are NTMs and why are they important?
- Simple physical picture of the instability
- Rutherford model equation
- Brief survey of exp'tal observation/ implications for ITER
- RF techniques of stabilization
- Role of rotation
- Outstanding theoretical and experimental issues.

Tearing Modes and Magnetic Reconnection

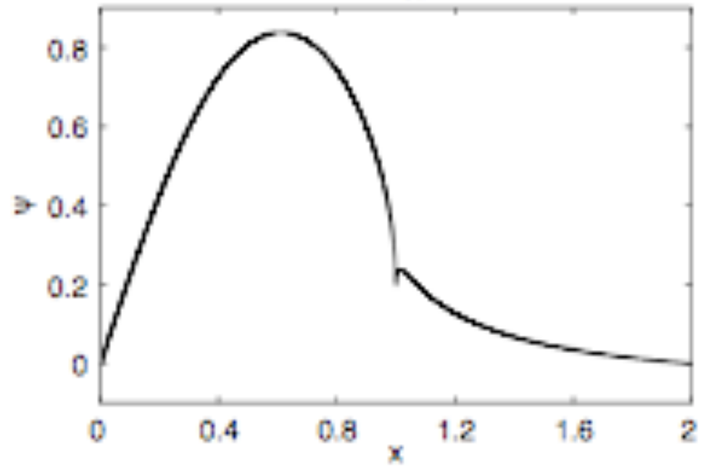
k•B=0



``Tearing'' of a current sheet

Classical Tearing Modes

- Asymptotic theory- uses two regions of the plasma
  - Outer region - marginal ideal MHD - kink mode
  - Inner region - include effects of inertia, resistivity nonlinearity, viscosity etc.



- Matching between inner and outer region

$$\frac{1}{2}\Delta'\psi_1 = \mu_0 R \int_{-\infty}^{\infty} d\rho \oint \frac{d\alpha}{2\pi}\cos(m\alpha) J_{\parallel},$$

- Linear theory : $\gamma \sim (\Delta')^{4/5} S^{-3/5}$

Magnetic island evolution in classical tearing modes

- Near mode rational surface **k·B = 0** ,

  $B_0 = B(r=r_s) - B_\theta(nq'/m)(r-r_s)\boldsymbol{\alpha}$, $\boldsymbol{\alpha} = \theta - (n/m)\varsigma$

  $\delta B = \delta B_r \sin(m\alpha)\, \mathbf{r}$

- Leads to the formation of a **magnetic island**

- Island width $w = 4(\delta B_r\, r_s / B_\theta\, nq')^{1/2}$

- when w > resonant layer thickness - nonlinear effects important

- Nonlinear evolution – Rutherford regime

$$\frac{dw}{dt} \approx \eta\Delta'$$

$$\Rightarrow w \propto t$$

What are NTMs?

- NTMs are relatively large size **magnetic islands** that develop slowly at mode rational surfaces with low (m,n) mode numbers in **high temperature tokamak** plasmas.

- Like the classical TMs they are current driven but the current source is the **bootstrap current** - a neoclassical (toroidal geometry driven) source of free energy.

- They limit the attainable β in a tokamak to values well below the ideal MHD limit - hence they are a **major concern** for all reactor grade machines i.e. long pulse (steady state) devices.

- Their temporal evolution is adequately modeled by a generalized form of the Rutherford Equation

Classical Tearing mode:

$$\boxed{E_{\|} = \eta J_{\|}}$$

$$E_{\|} \sim -\frac{\partial A_{\|}}{\partial t} \qquad J_{\|} \sim -\nabla^2 A_{\|}$$

$$\Downarrow \qquad \Downarrow$$

$$\frac{d\delta B}{dt} = \eta \frac{\Delta'}{w} \delta B \quad \Rightarrow \quad \boxed{\frac{dw}{dt} \approx \eta \Delta'}$$

- **In high temperature tokamaks neoclassical effects need to be retained**

Modified Ohm's Law

$$< E_{\parallel} > \; = \; \eta J_{\parallel} + \frac{1}{neB} < B \cdot \nabla \cdot \pi_{\parallel e} >$$

⇓

Bootstrap current

⇑

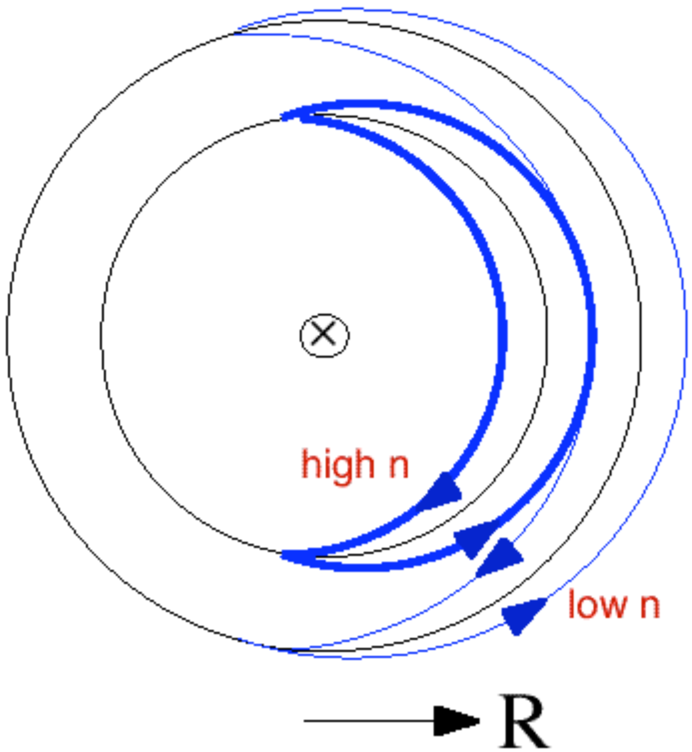
$$\frac{1}{neB} < B \cdot \nabla \cdot \pi_{\parallel e} > \; \approx \; \frac{\mu_e}{\nu_e} \frac{1}{B_\theta} \frac{dp}{dr} + \eta \frac{\mu_e}{\nu_e} J_{\parallel}$$

Electron viscous stress which describes damping of poloidal electron flows - new free energy source.

Dependence on pressure gradient, also fraction of trapped particles

BOOTSTRAP CURRENT

Projection into a poloidal plane



generated by trapped particles:

example: banana particles

- electrons drift from flux surfaces due to the ∇B-drift
- electrons with low parallel velocity are trapped in the toroidal mirror
  $\Rightarrow$ **banana orbits**
- at the intersection of 2 banana orbits a net current results due to the density gradient
- passing particles exchange momentum with trapped particles
  $\Rightarrow$ **bootstrap current**

similar: helically trapped particles

Modified Rutherford Equation

$$\frac{dw}{dt} = \frac{\eta}{\mu_0}(\Delta' + \frac{D_{nc}}{w})$$

where $$D_{nc} = -\sqrt{\epsilon}\,\frac{2\mu_0}{B_\theta^2} p' \frac{q}{q'} k_0$$

$$p'q' < 0, \quad D_{nc} > 0$$

Unstable for normal tokamak operation

$$p'q' > 0, \quad D_{nc} < 0$$

Stable in reversed shear regions

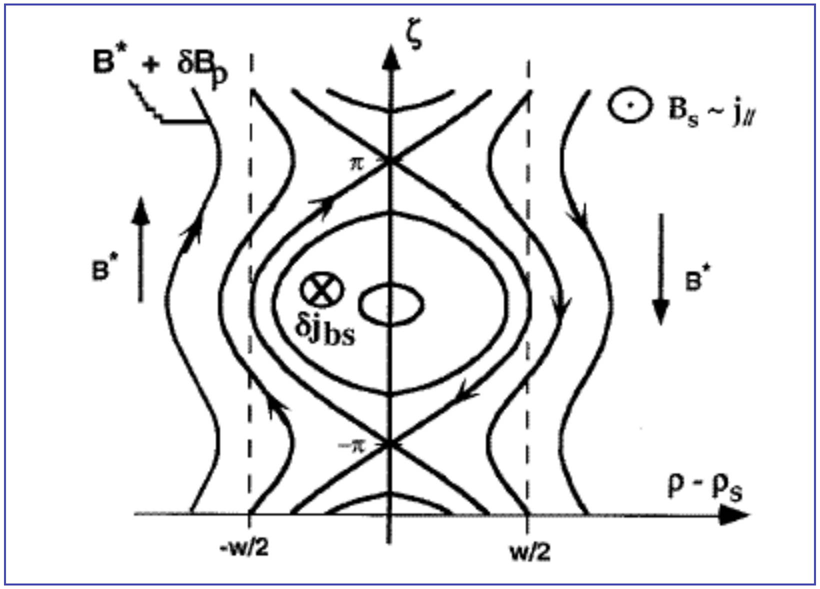
- Can be unstable for $\Delta' < 0 \Rightarrow$ $$w_{sat} = \frac{D_{nc}}{-\Delta'} \approx \frac{r_s \beta_\theta}{m}$$

- for small islands $$w \sim \sqrt{\eta t}$$

PHYSICS OF NTM

- Plasma pressure profile is flattened within the island - J_{bs} **is turned off**
- **This triggers a δJ_{bs} with the same helical pitch as the island**

- the corresponding induced δB has the same direction as the initial perturbation and **enhances it**



This picture neglects finite perpendicular thermal conductivity within the island - important for small island widths - leads to **threshold size**.

Finite perpendicular thermal conductivity effect

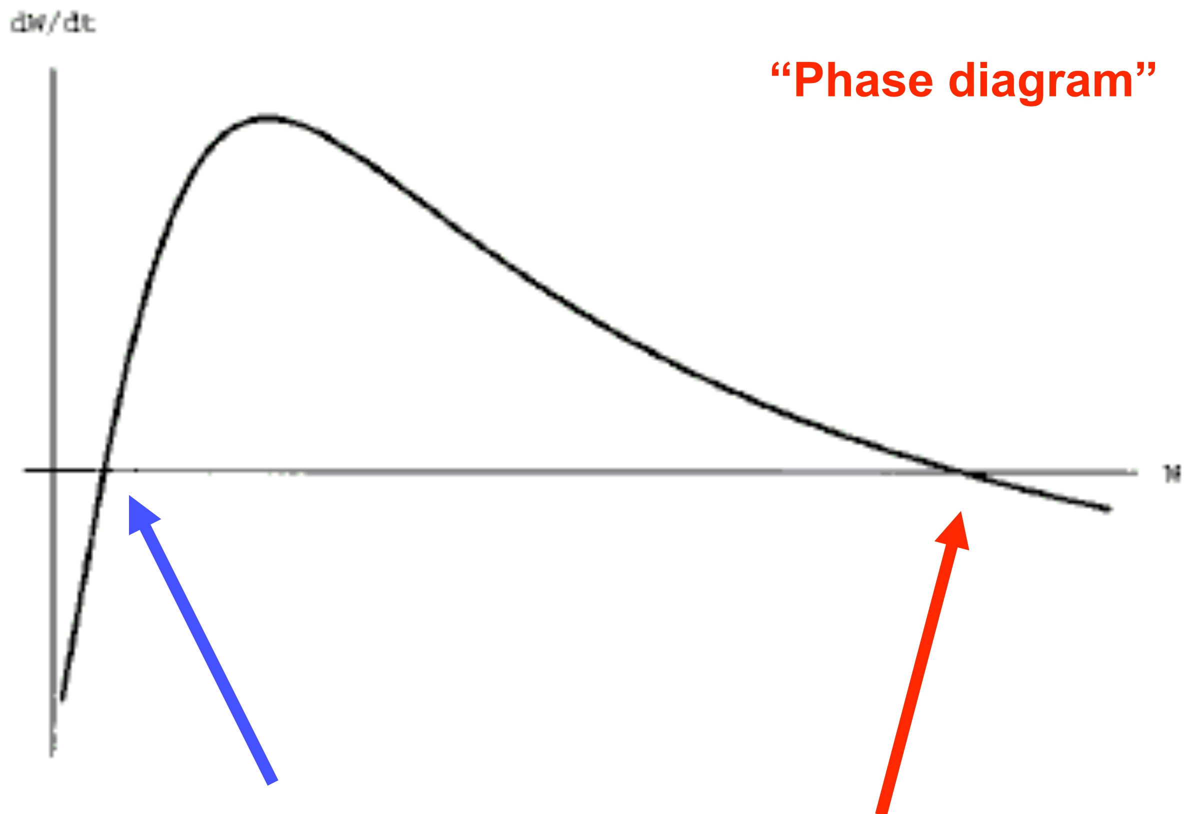
$$\frac{dw}{dt} = \frac{\eta}{\mu_0}(\Delta' + D_{nc}\frac{w}{w^2 + w_c^2})$$

$$w_c \sim \left(\frac{\chi_\perp}{\chi_\parallel}\right)^{1/4} \sqrt{\frac{q^2 R}{mq'}}$$

Threshold - “seed” – island size

$$w_{seed} = -\frac{\Delta' w_c^2}{D_{nc}}$$

NTM characteristics



"seed" island necessary for growth
– so NTM is a nonlinear mode "subcritical instability"

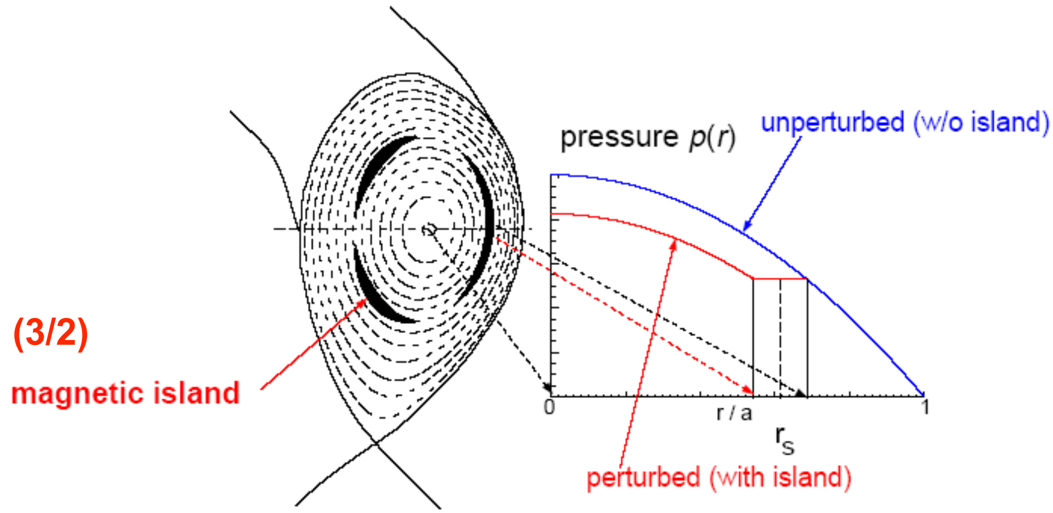
How is the seed island created?

Saturation width proportional to β_θ - hence limits plasma pressure

Effects of NTMs

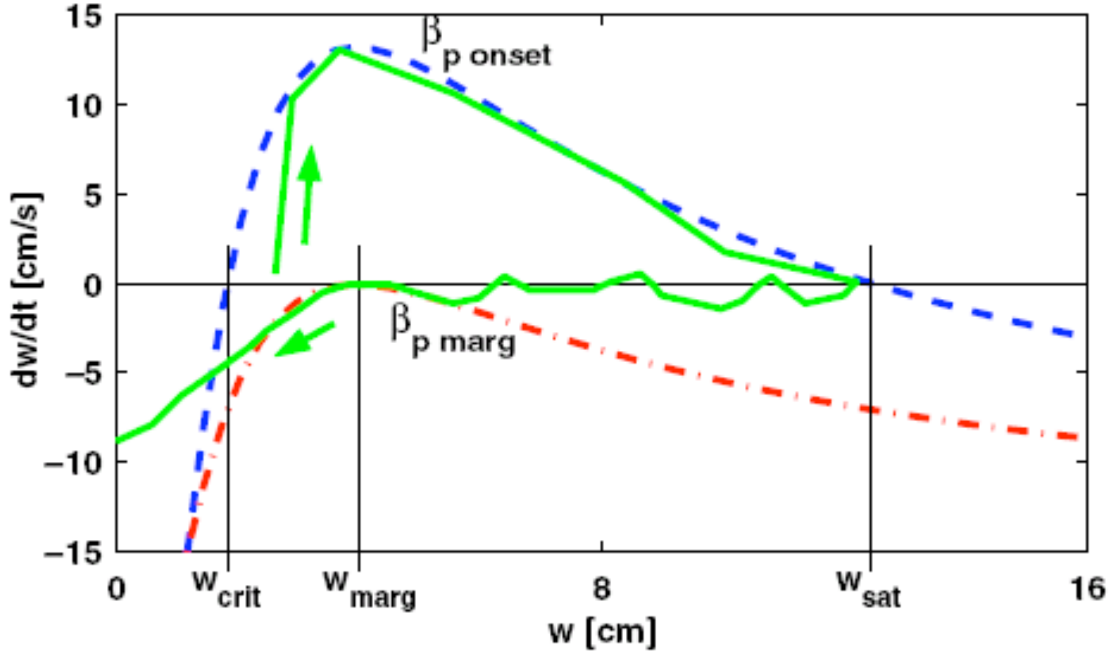
• **Can degrade confinement** – fast temperature flattening across island due to high parallel thermal conductivity

$$\frac{\Delta\tau_E}{\tau_E} = 4\frac{w\rho_s^3}{a^4}$$



• Can cause **disruption** if island size becomes comparable to distance between mode rational surface and plasma edge (depends on beta_poloidal)

Time evolution of an NTM growth rate



Brief Survey of Experimental Observations on NTMs

Experimental observation of NTMs

- Earliest observations were on TFTR - in supershot discharges

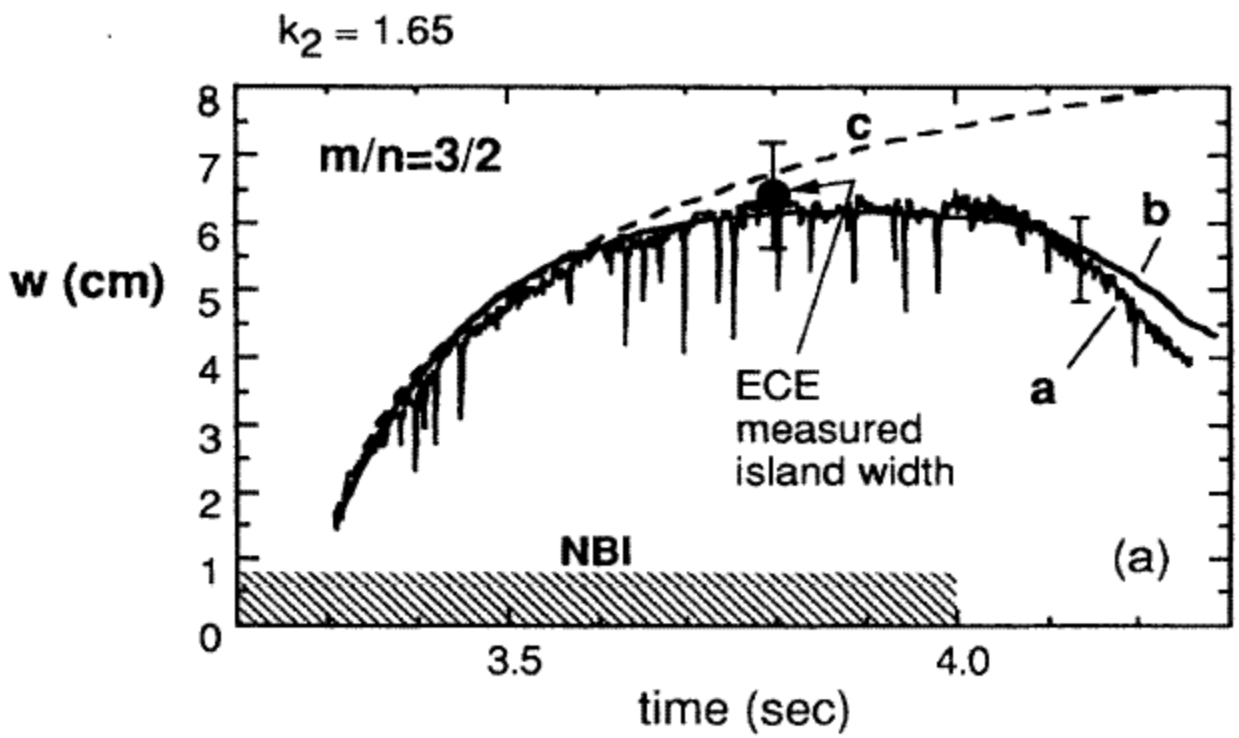
- **Mainly (3/2) or (4/3) modes with f<50khz**

- Degradation of plasma performance with growth of NTM

- Characteristics agreed quite well with Rutherford model estimates

(Z. Chang et al, PRL **74** (1995) 4663)

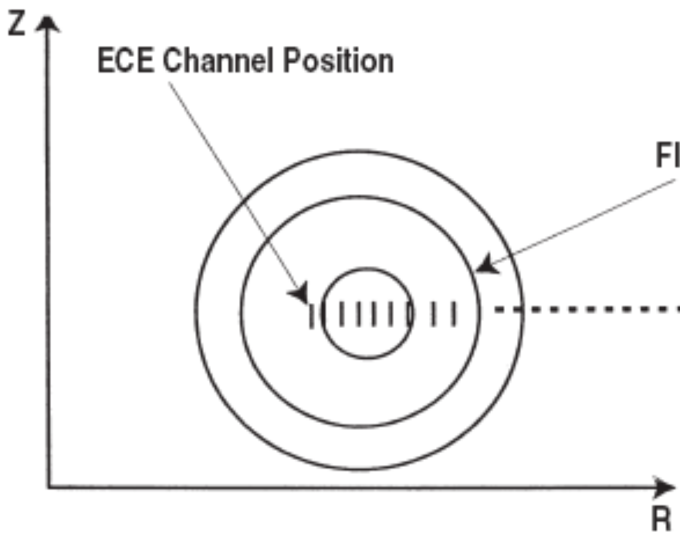
TFTR



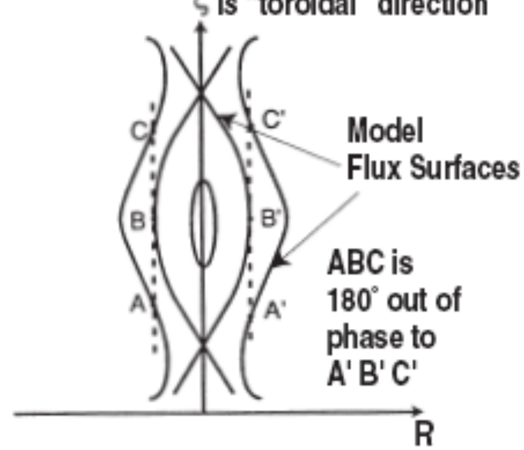
Comparison of “measured” island widths with Rutherford model estimates.

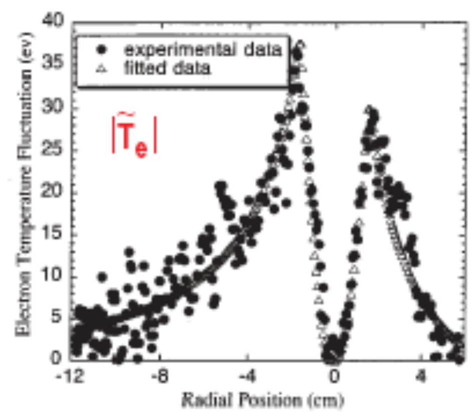
Island Structure Can be Measured by Electron Cyclotron Emission of T_e Fluctuation Radial Profile

- Magnetic surface distortion
  - ★ leads to T_e fluctuation



(Y. Nagayama et al., 1990)





(C. Ren, et., 1998)

w also measured by magn. Probes:

$$w = 4\sqrt{\frac{q\psi}{q' B_{\theta_s}}} = 4\sqrt{\frac{R_0 q}{B_0 s}\rho_s^m \delta B_{\theta,mn,edge}}.$$

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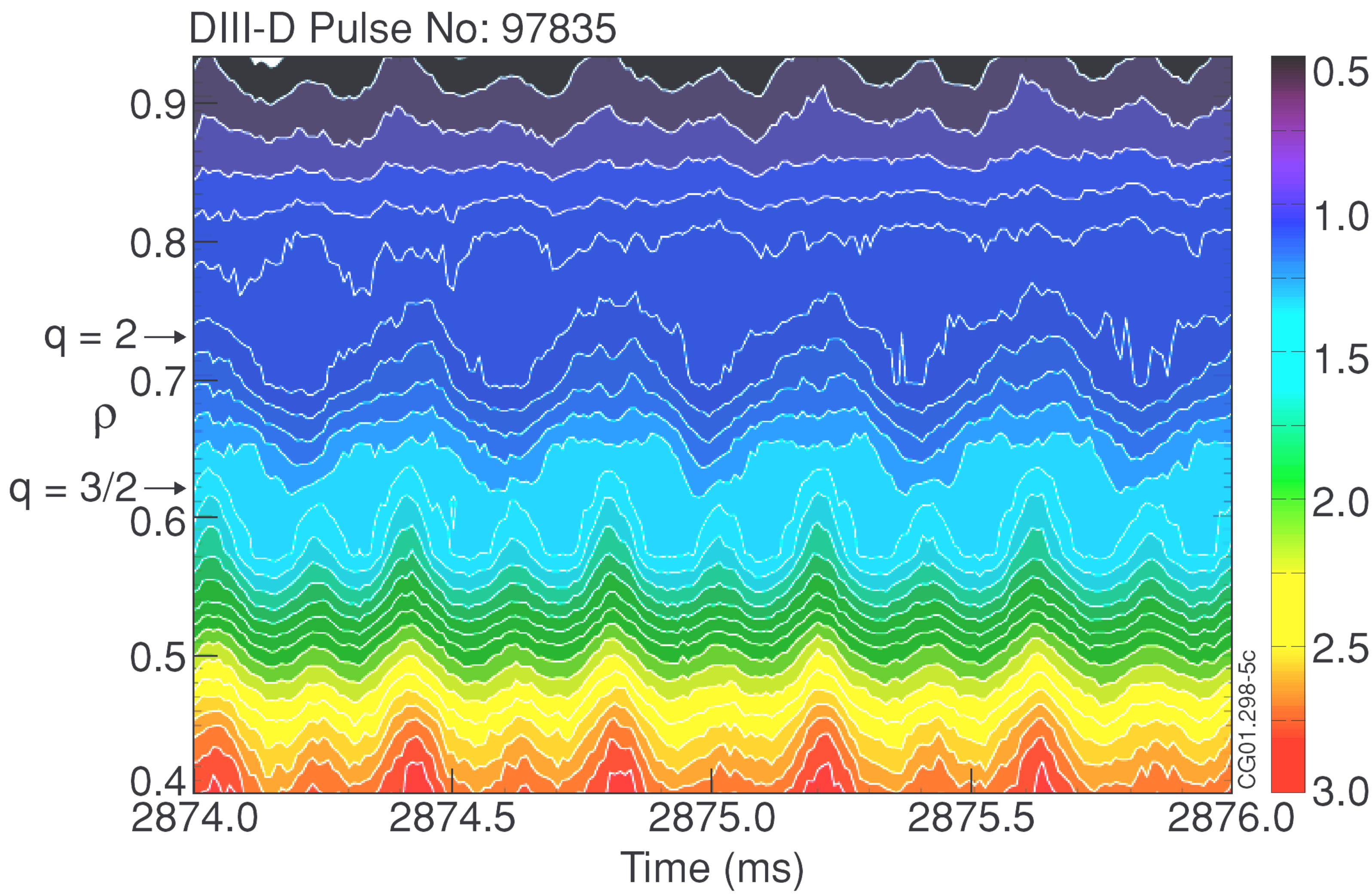
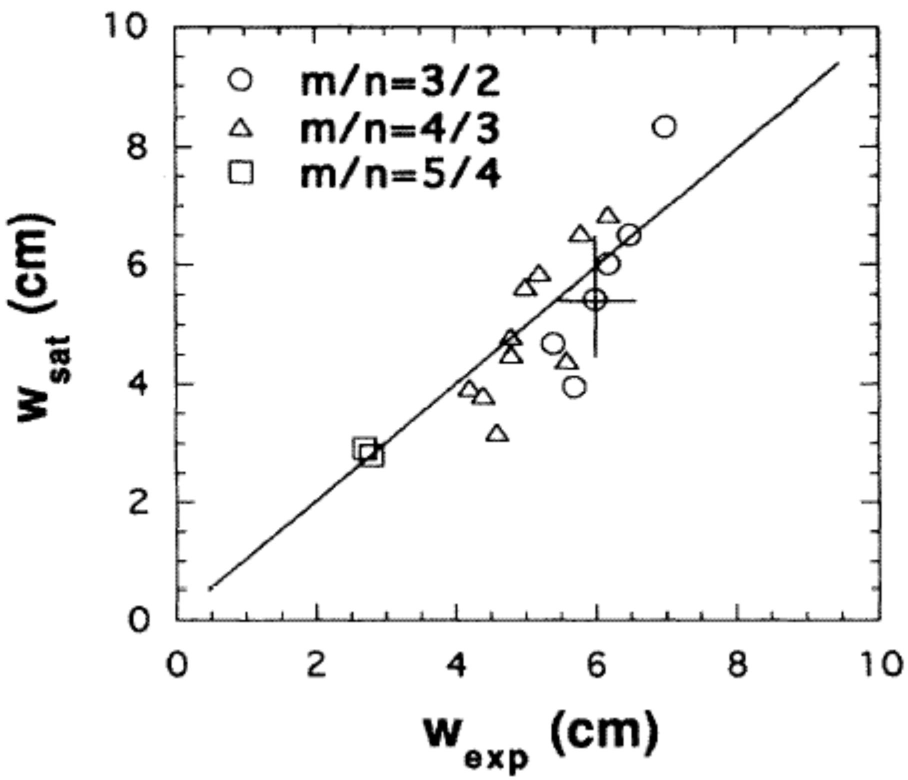


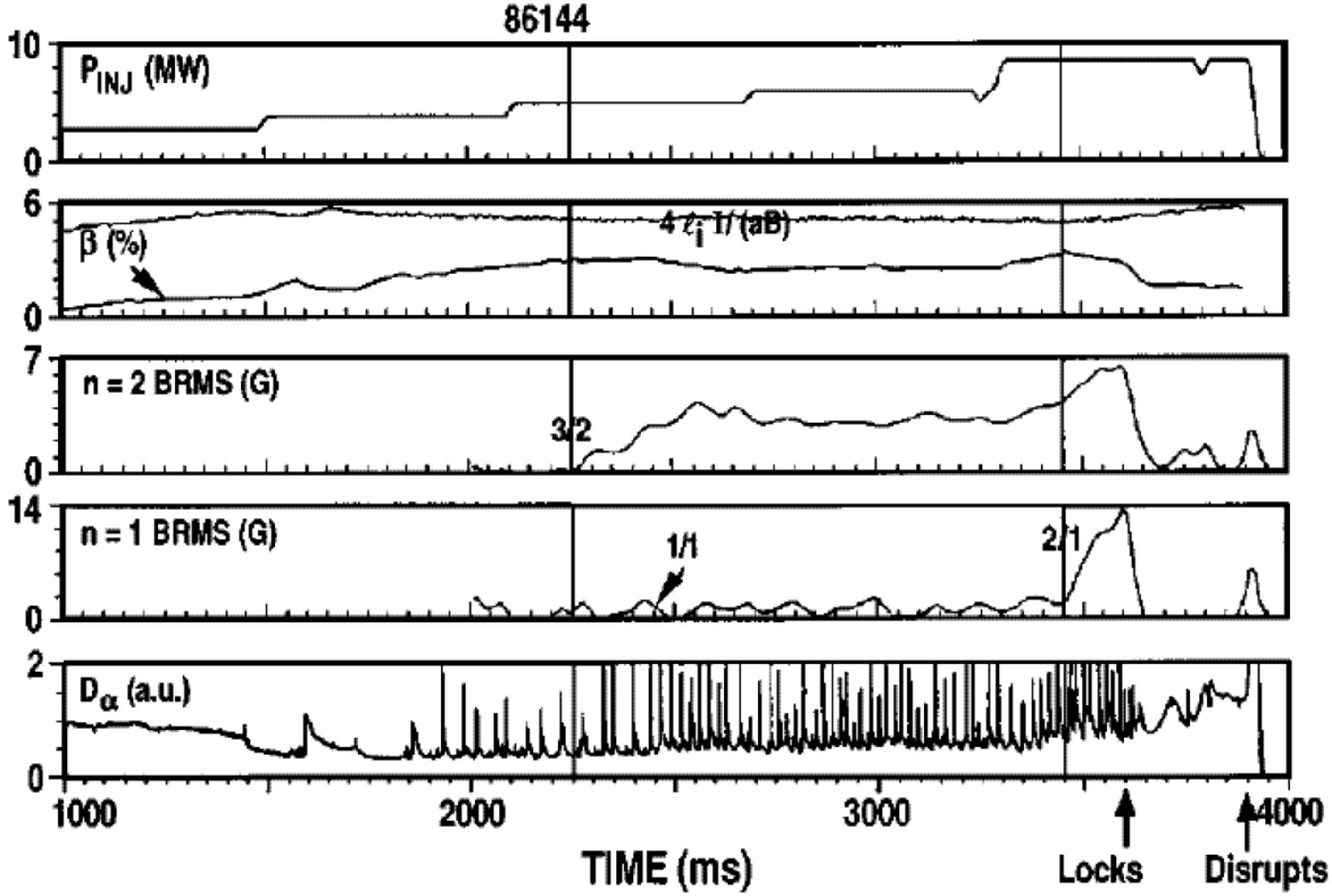
T Hender et al, Nucl Fus 2002

TFTR



Theory - experiment comparison of saturated island widths

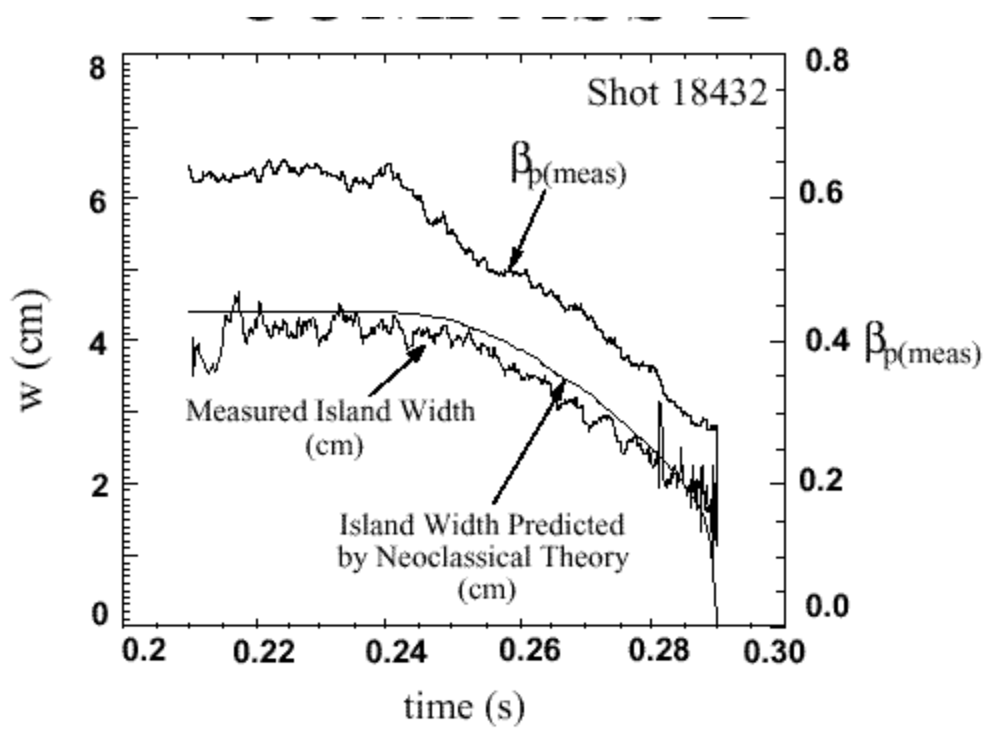
D- III- D observations



A 3/2 mode is excited at t=2250 - saturates beta; at t=3450 a 2/1 mode grows to large amp, locks and disrupts. Ideal beta limit is 3.4

[O. Sauter et al, PoP 4 (1997) 1654]

COMPASS D

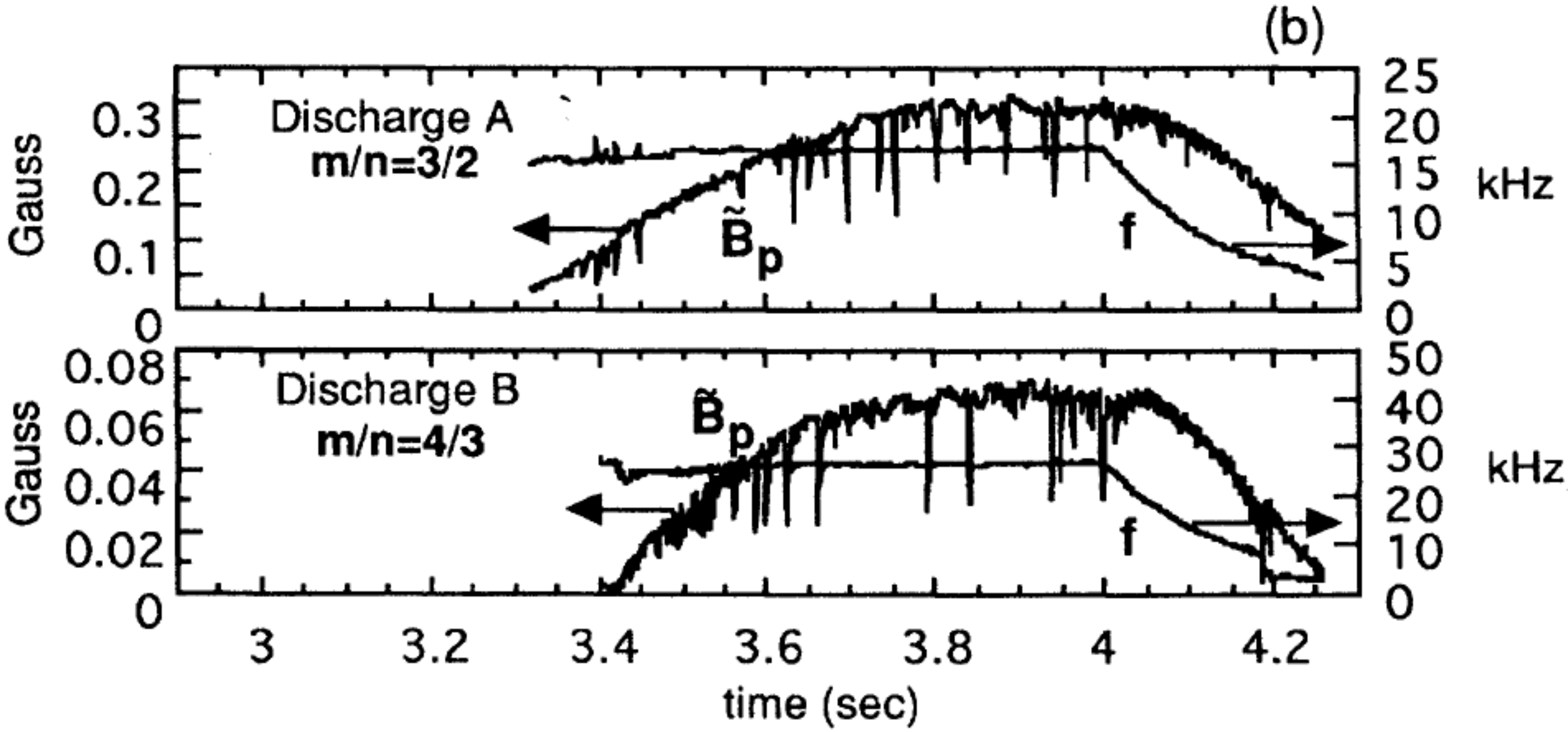


Saturated island width scales like β_p

$$w_{sat} = -a_1 \varepsilon^{1/2} \left(\frac{L_q}{L_p} \right) \frac{\beta_p}{\Delta'}$$

[D.A. Gates et al, Nuclear Fusion **37** (1997) 1593]

TFTR



Single helicity NTMs; f<50 kHz

ASDEX UPGRADE

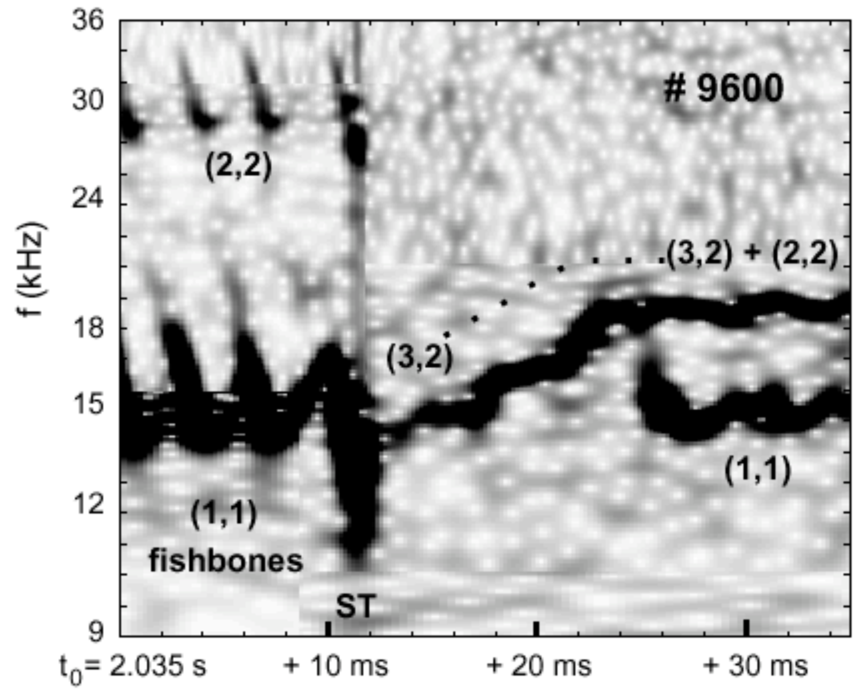


Figure 3. Wavelet plot of an early NTM immediately after a sawtooth crash. The NTM frequency rises during the first 10 ms.

Many experiments have shown a strong correlation between a sawtooth crash and an NTM excitation

ASDEX UPGRADE

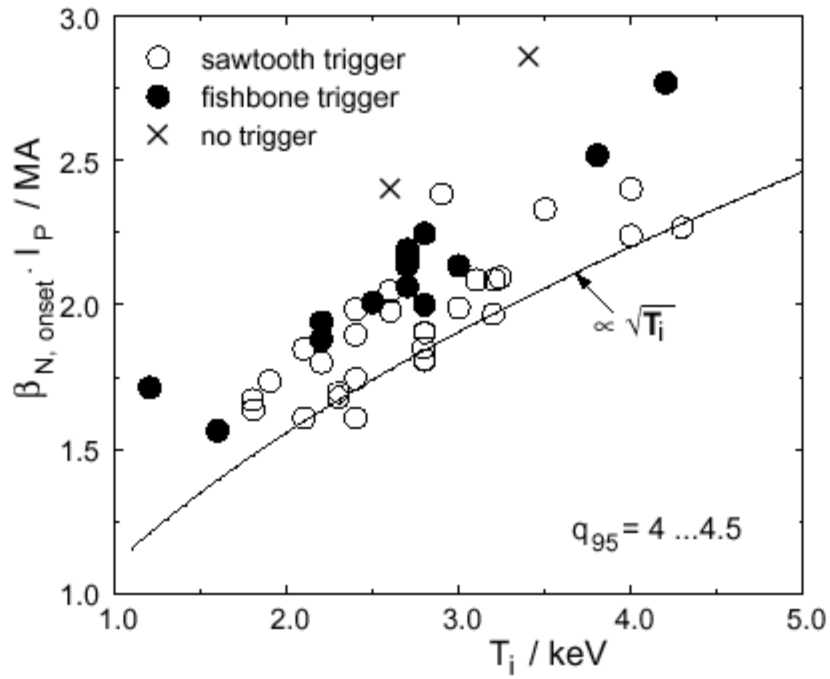


Figure 4. $\beta_{N,onset} \cdot I_p$ vs. the ion temperature at the $(3, 2)$ radial position, T_i. Additionally the scaling, $\beta_{N,onset} \cdot I_p \propto \sqrt{T_i}$, is shown [2].

ASDEX U

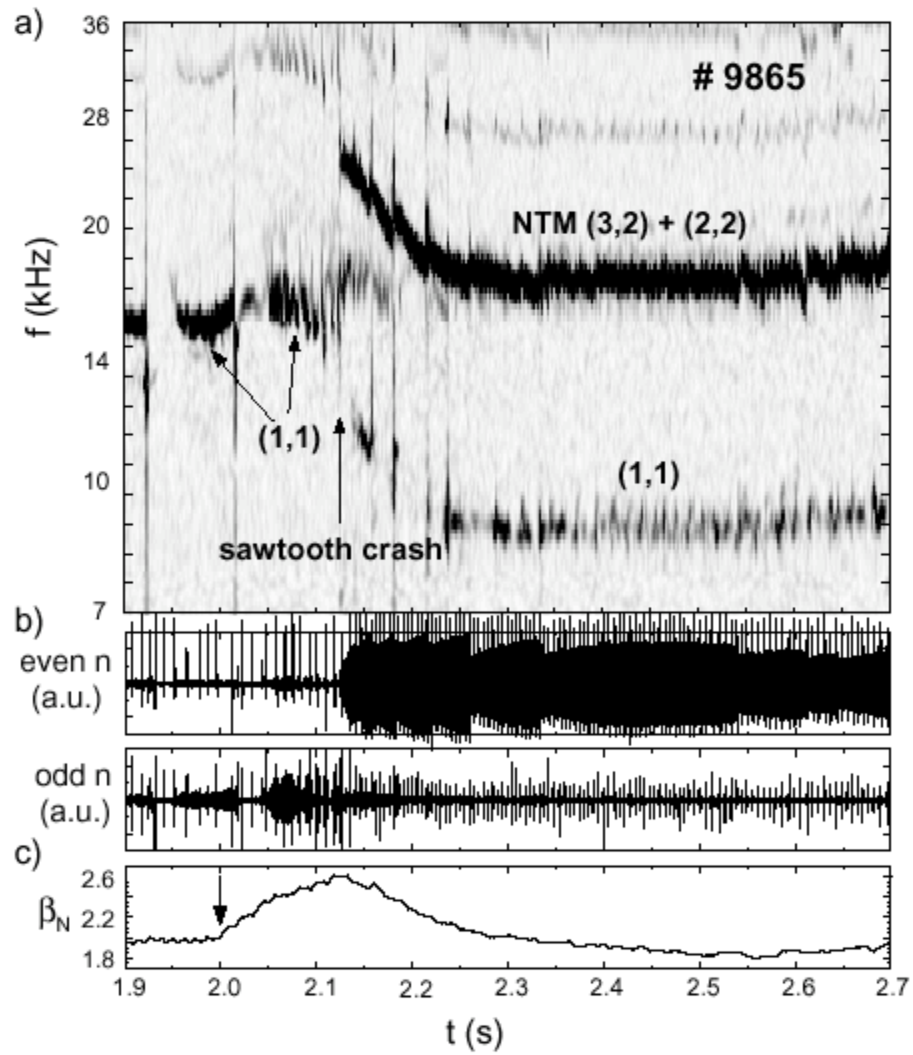
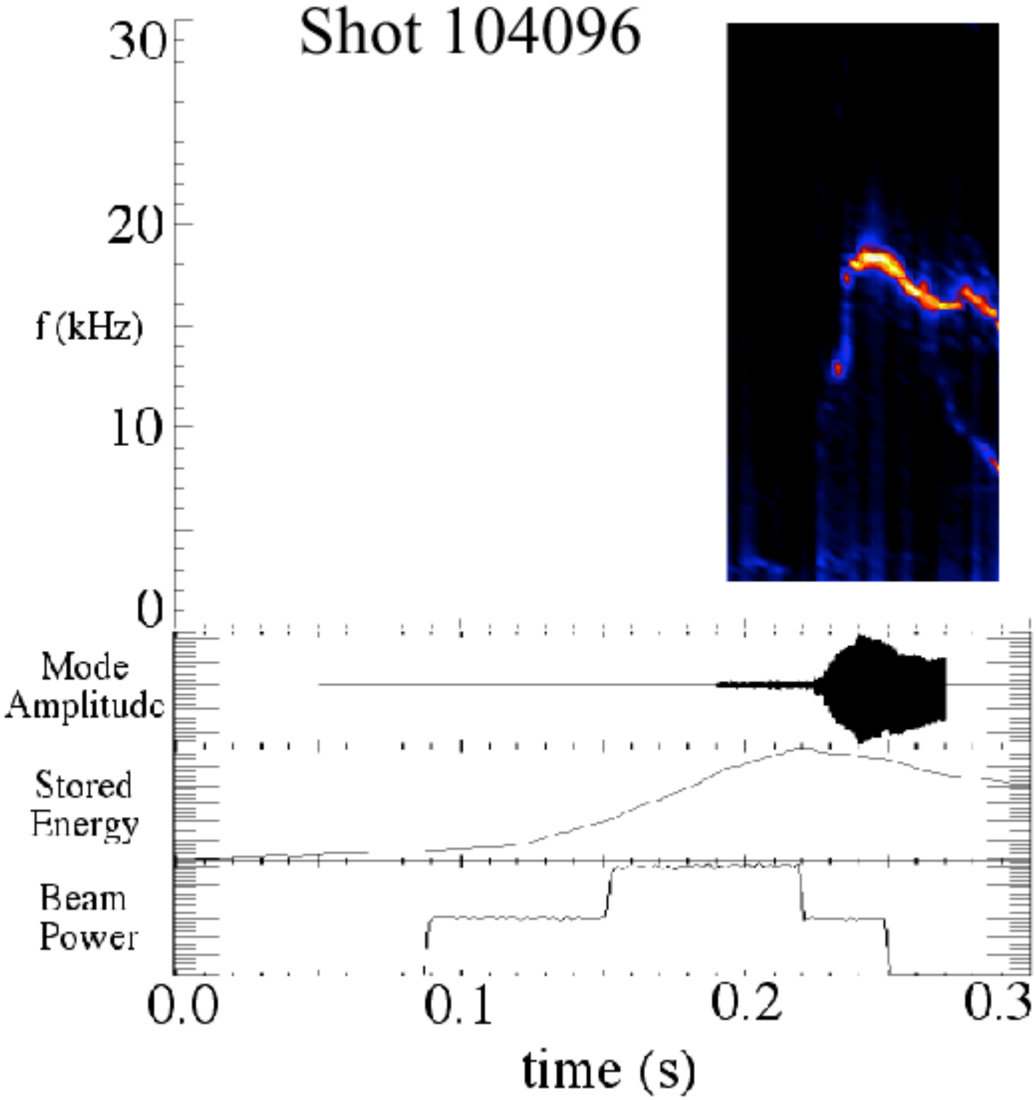


Figure 1. a) Wavelet plot [6] of an NTM. Dark areas represent mode activity. Before the onset of the NTM at 2.126 s fishbone bursts are seen. b) Mirnov signals. The even n signal is dominated by the NTM, the odd n signal by (1,1) modes. c) $\beta_N = \beta_t aB/I$ with $\beta_t = 2\mu_0 p/B_t^2$; the arrow indicates the increase of neutral beam injection power from 5 to 7.5 MW.

NTMs can also be triggered by fishbone activity
Other triggers: ELMs….

NSTX



- Mode appears at constant poloidal β (β_p ~0.4)
- Slower growth ⇒ resistive mode
- Beam turn off experiment indicates amplitude reduction with stored energy
  - *indicative of bootstrap current driven tearing mode*

How to eliminate or control NTMs?

- Directly control NTMs through appropriate feedback control schemes
  - **ECCD** scheme most successful
  - Also **ECH**
- Get to the trigger : prevent sawtooth crash, prevent large ELMs etc
- Other ideas: profile control, rotation, mode coupling etc

How to Stabilize an NTM?

- Principal Idea: **Restore the suppressed bootstrap current within the island**

  - localized current drive -- ECCD, LHCD, NB(?)

  - localized heating - helical temperature variations modify current profile

  - localized density deposition - also changes pressure

- Ohm's law with auxiliary current

$$J_{\|}(\Psi) = \frac{1}{\eta}\langle E_{\|}\rangle + \frac{1}{\eta B}\langle \mathbf{B}\cdot\nabla\cdot\pi_{\|e}\rangle + \langle J_{\mathrm{aux}}\rangle,$$

- **Modified Rutherford Equation**

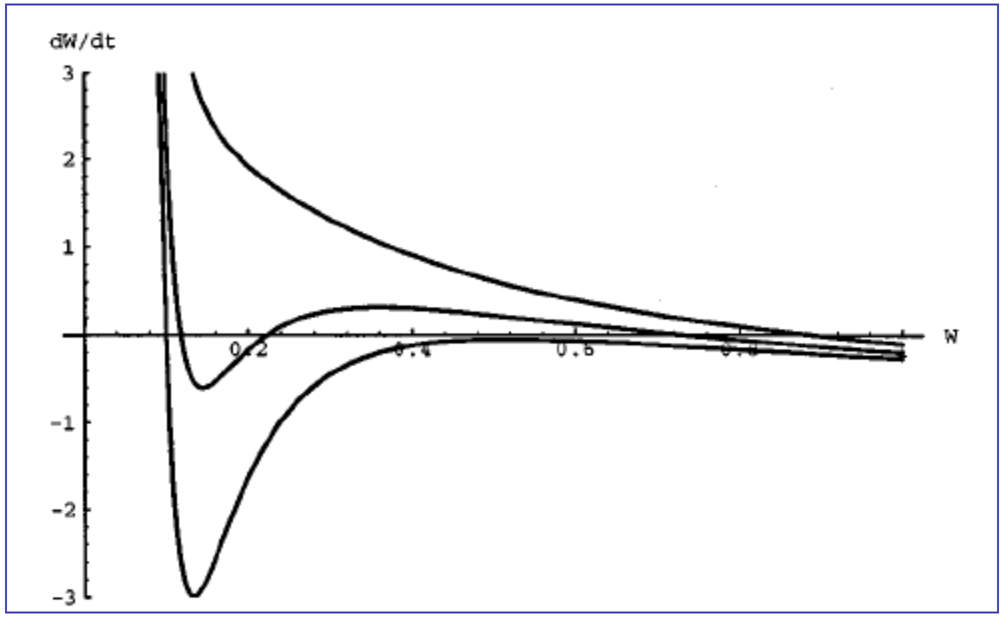
$$0.82\,\frac{dw}{dt} = \frac{1}{\tau_r}\left(\Delta'\rho_s + \frac{D_{nc}}{w} - \frac{D_{\mathrm{aux}}}{w^2}\,\eta_{\mathrm{aux}}\right),$$

$$D_{\mathrm{aux}} = \frac{I_{\mathrm{aux}}\mu_0 R}{s\,\psi_s'\rho_s}\,\frac{16}{\pi},$$

η_{aux} is an efficiency factor

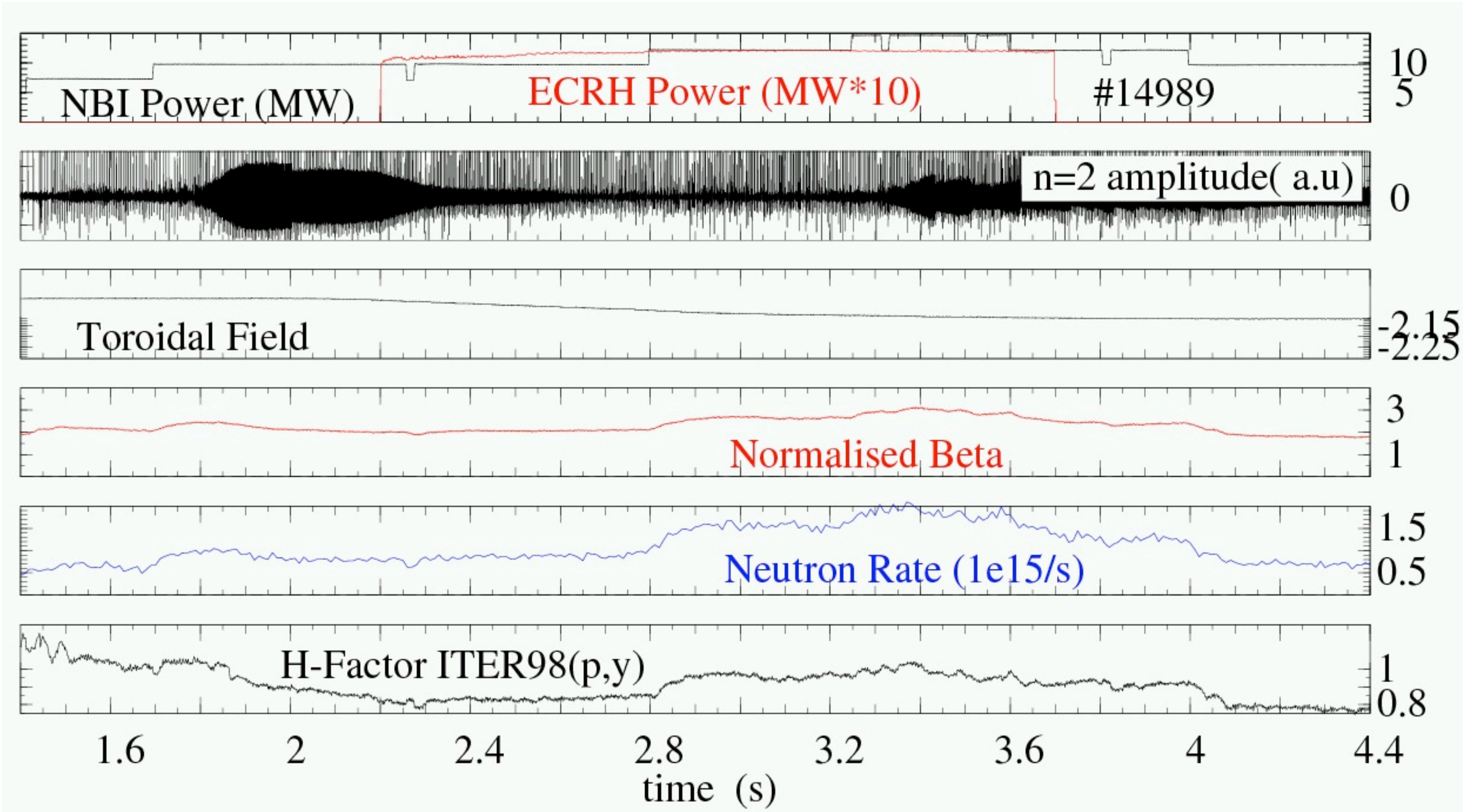
New "phase diagram"

- Stable and unstable fixed points corresponding to saturated island sizes



$$\eta_{\mathrm{aux}} D_{\mathrm{aux}} > \frac{1}{4} \frac{(D_{nc})^2}{(-\Delta' \rho_s)},$$

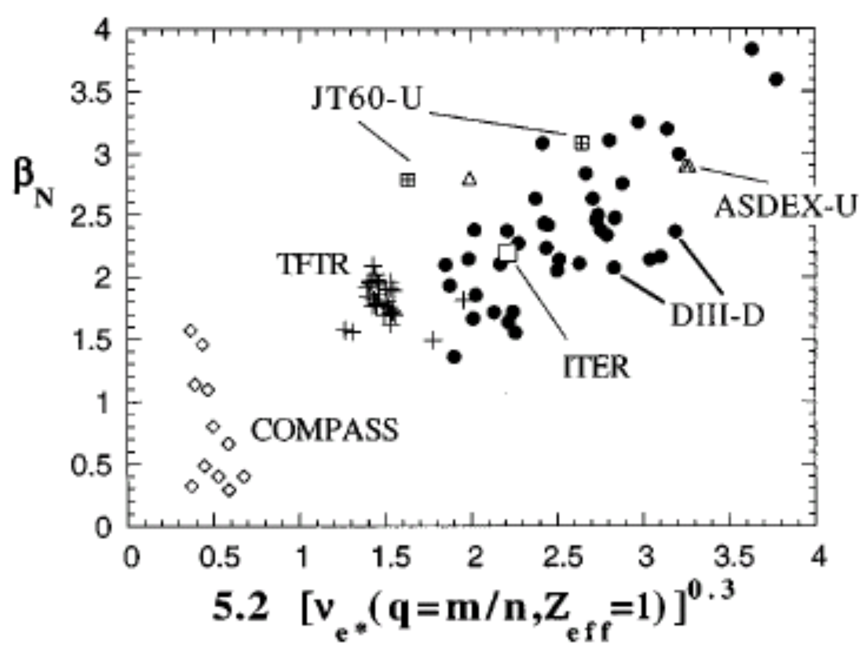
Condition for complete stabilization



Complete stabilization of a 2/1 NTM in ASDEX-U

Implications for ITER

- Seed island size ~ 5 to 6 cms
- Saturated island size can be about 60 cms limiting β_N ~ 2.2
- Growth time - 30 s to reach 30 cms & about 150 s to reach 60 cms
- Based on modeling and extrapolation from experiments simulating the ITER parametric regime



Local Heating Effects

$$\delta J_{\|} = \frac{3}{2}\frac{\delta T_e}{T_{eo}} J_{\|o},$$ helically resonant temperature variations

$$0.82\frac{dw}{dt} = \frac{1}{\tau_r}\left(\Delta' \rho_s + \frac{D_{nc}}{w} - w D_{\text{heat}}\right),$$

$$D_{\text{heat}} = \frac{16}{5\pi}\frac{q_s}{q_s'}\frac{R\mu_o J_{\|o}}{\psi_s'}\frac{S_o \rho_s^2}{n T_e \chi_\perp}$$

$$w_{\text{sat},H} = \frac{D_{nc}}{-\Delta' \rho_s}\frac{2}{1+\sqrt{1+\Upsilon}},$$

Demonstrated in TEXTOR – complete stabilization of 2/1 mode

E. Westerhof et al, NF 47 (2007) 85

Sen, Kaw and Chandra - IAEA, '98 – NF 2000

- ECRH scheme - self-consistent bootstrap currents created by the driven pressure gradients within the island can provide additional stabilization.

$$0.82\frac{dw}{dt} = \frac{1}{\tau_r}(\Delta'\rho_s + \frac{D_{nc}}{w} - wD_{heat} - wD_{bs}) \qquad D_{bs} = 0.14\sqrt{\epsilon}\frac{\mu_o\rho_s^2 R^2}{\psi_s'^2}\frac{q_s}{q_s'}\frac{S_{T0}}{\chi_\perp}\frac{\iota_o''}{\iota_o'}$$

Asymmetry in the island shape makes these currents important

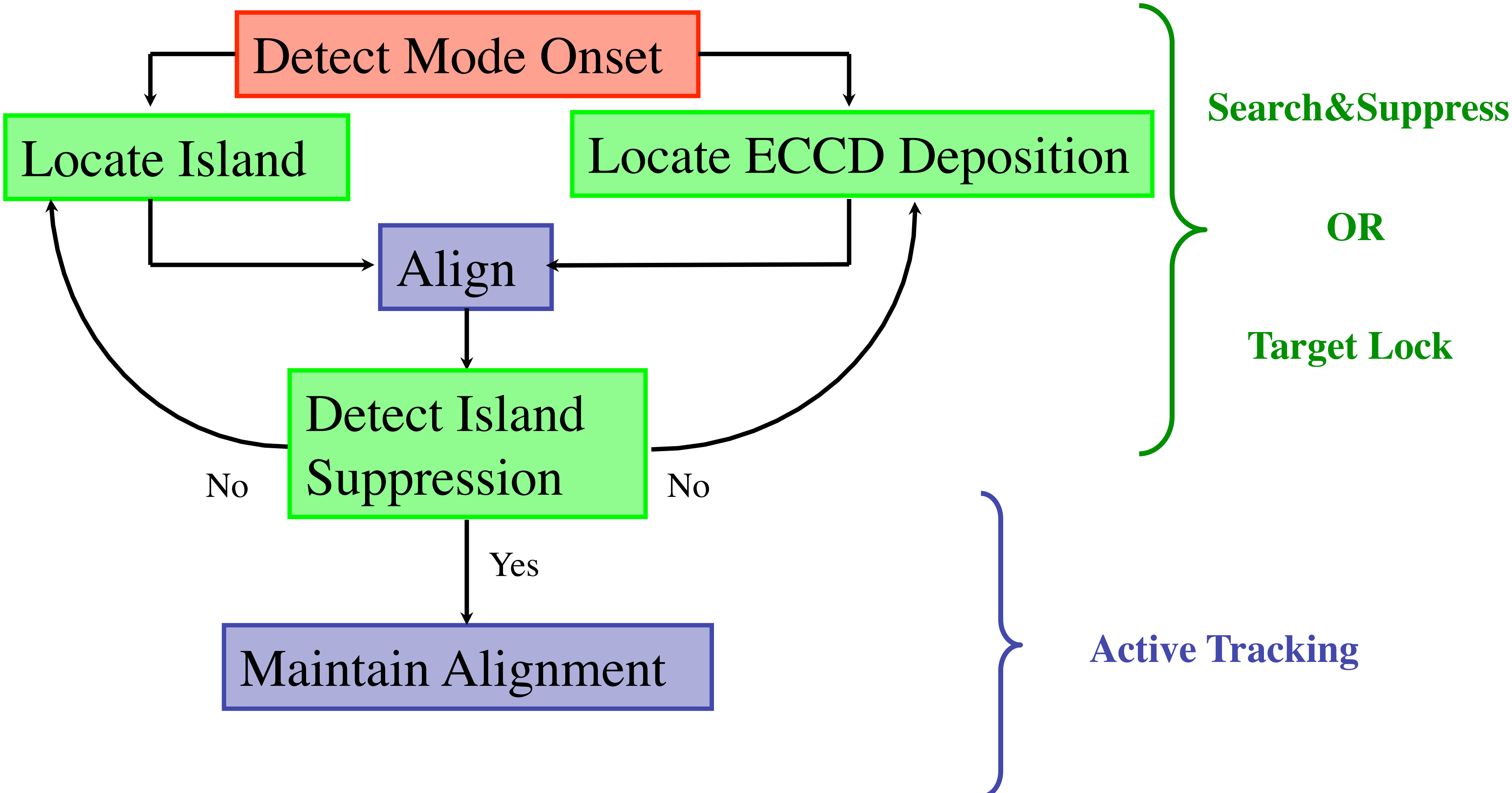
- Similar currents can arise from deposition of density or momentum within the island e.g. through neutral beams - new stabilization scheme proposed

- **Feedback suppression of NTMs using modulated neutral beams**

- **Beam power and energy requirements are quite realistic and achievable.**

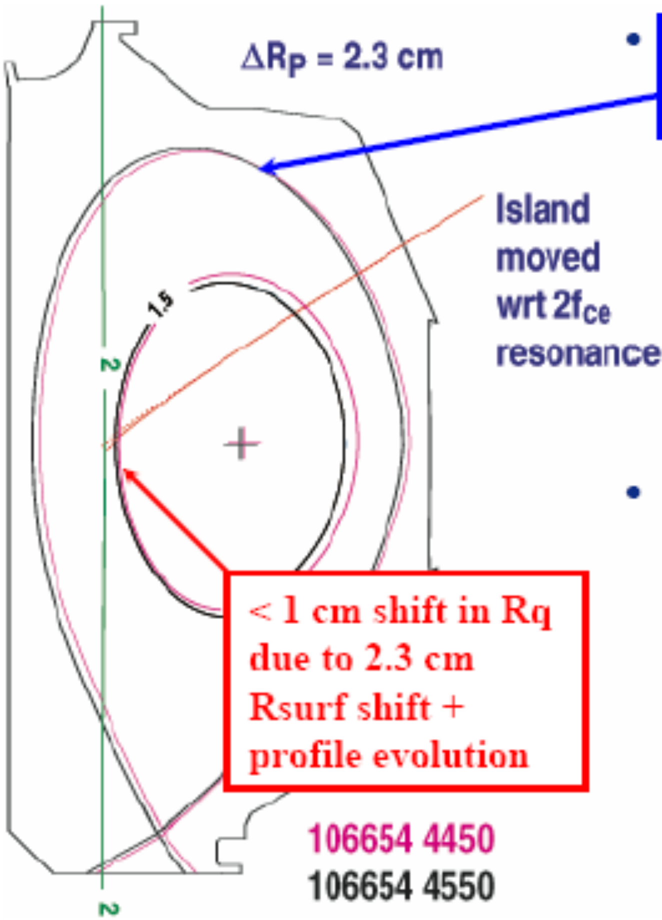
A. Sen, D. Chandra and P.Kaw, Nucl. Fus. 40 (2000) 707

NTM Control Requires Achieving and Sustaining Dynamic Island/ECCD Alignment

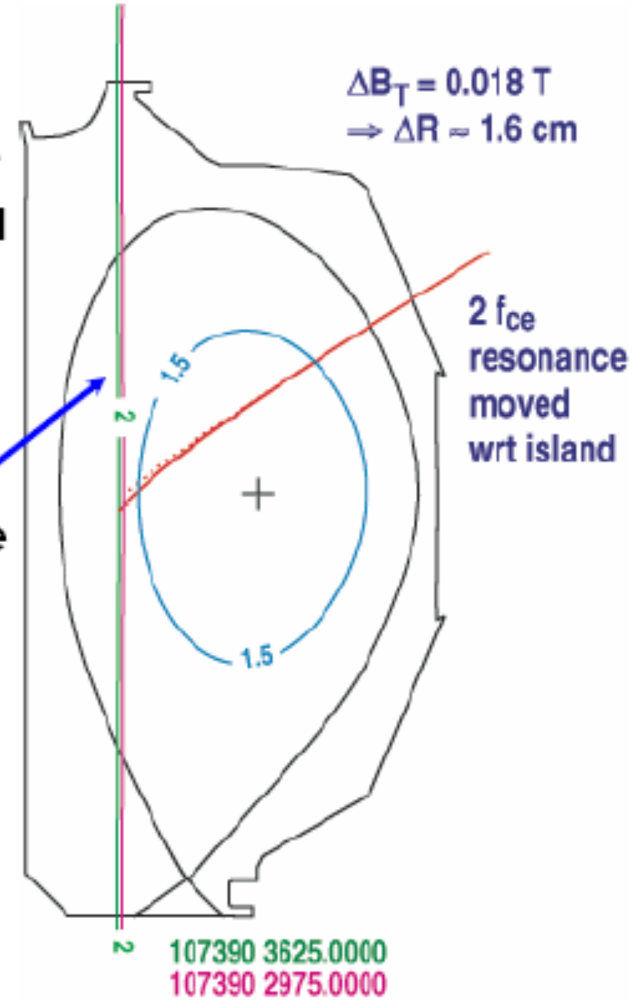


D. Humphreys, R. La Haye

Actuators: Variation of Plasma Position or Toroidal Field Are Used to Regulate Alignment

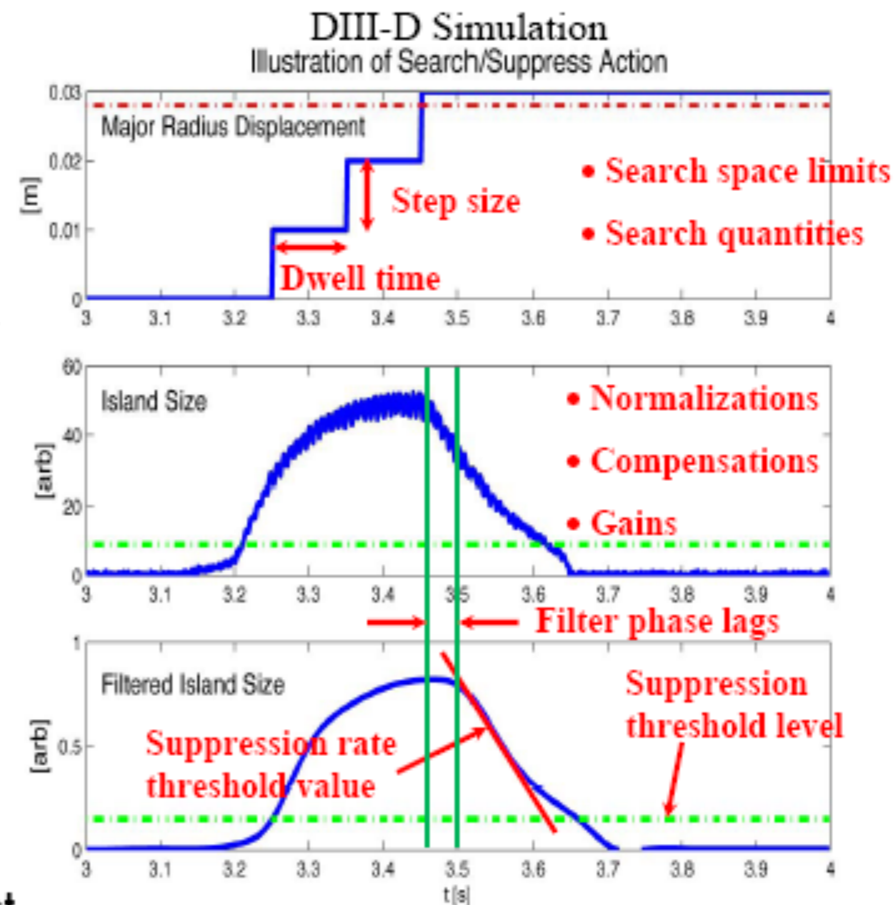


- Plasma surface major radius can be moved to shift island relative to ECCD
- Toroidal field can be varied to shift ECCD relative to island



"Search and Suppress" Algorithm Uses Island Response to Detect Island/ECCD Alignment

- **Uncertainty in locations of both island and ECCD comparable to alignment accuracy required (~ 1 cm) ⇒ need systematic search**
- **"Search and Suppress" algorithm:**
  - Vary alignment in steps (e.g. plasma major radius ΔR or toroidal field ΔB_T)
  - Dwell for specified time to measure island response
  - Freeze if island suppressed
- **Adjustable feedback parameters include filters, compensation for plasma motion and rotation**
- **Actuator limits prevent plasma-limiter contact**

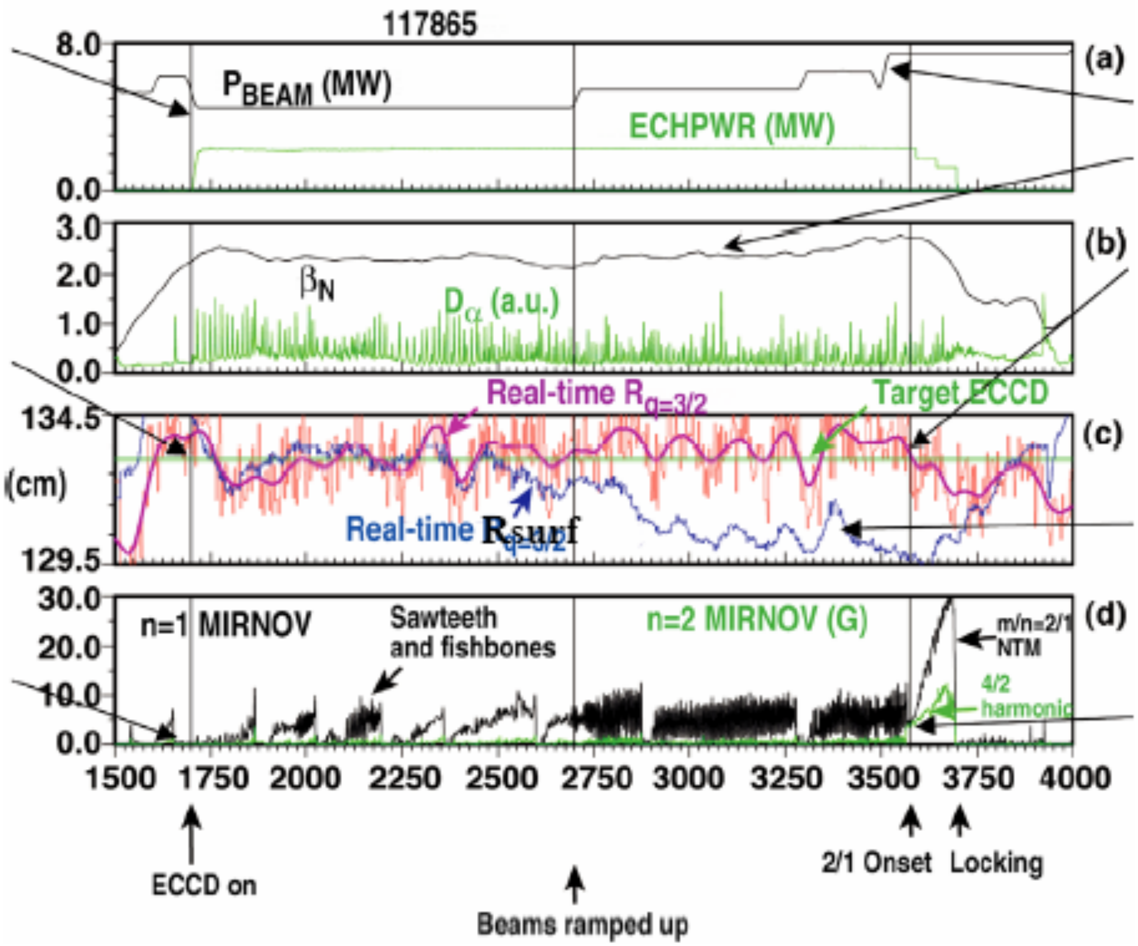


Active Tracking of q-Surface Motion Enables Preemptive NTM Suppression

ECCD and control enabled

ECCD initially ~ aligned with q=3/2 surface

No NTM initially

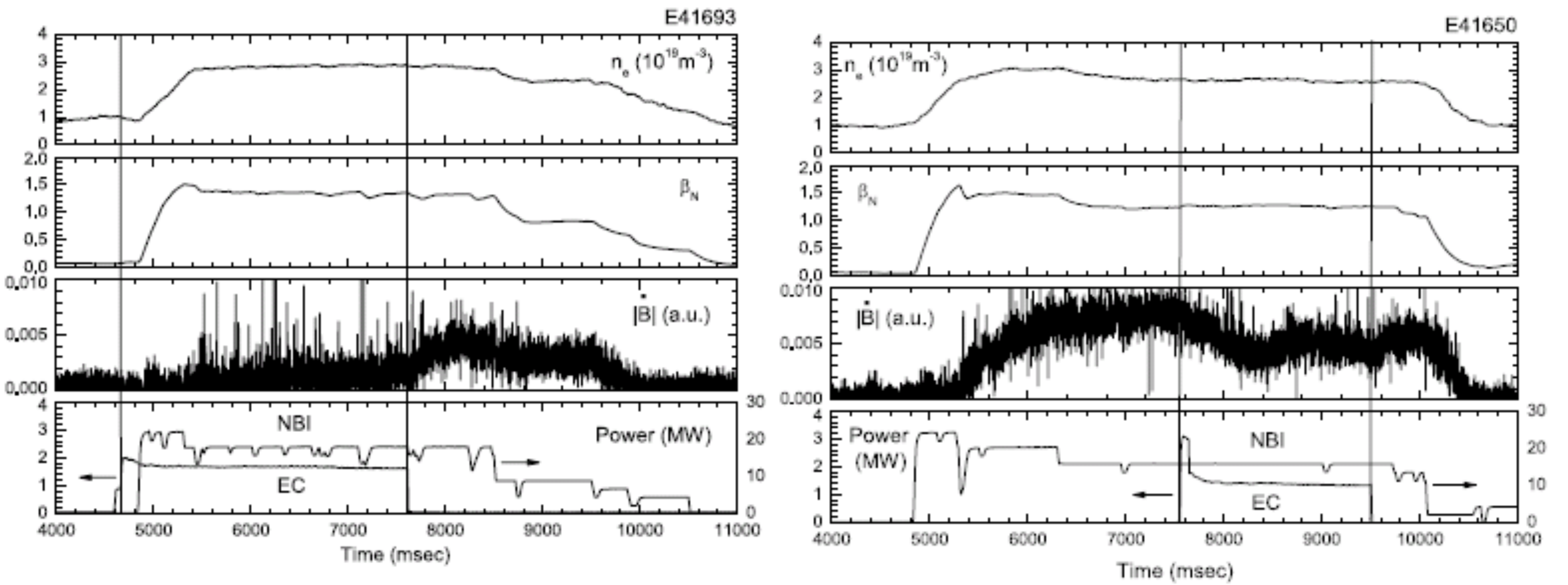


Beam power and β_N are increased…

..alignment is maintained with realtime q=3/2 surface reconstruction..

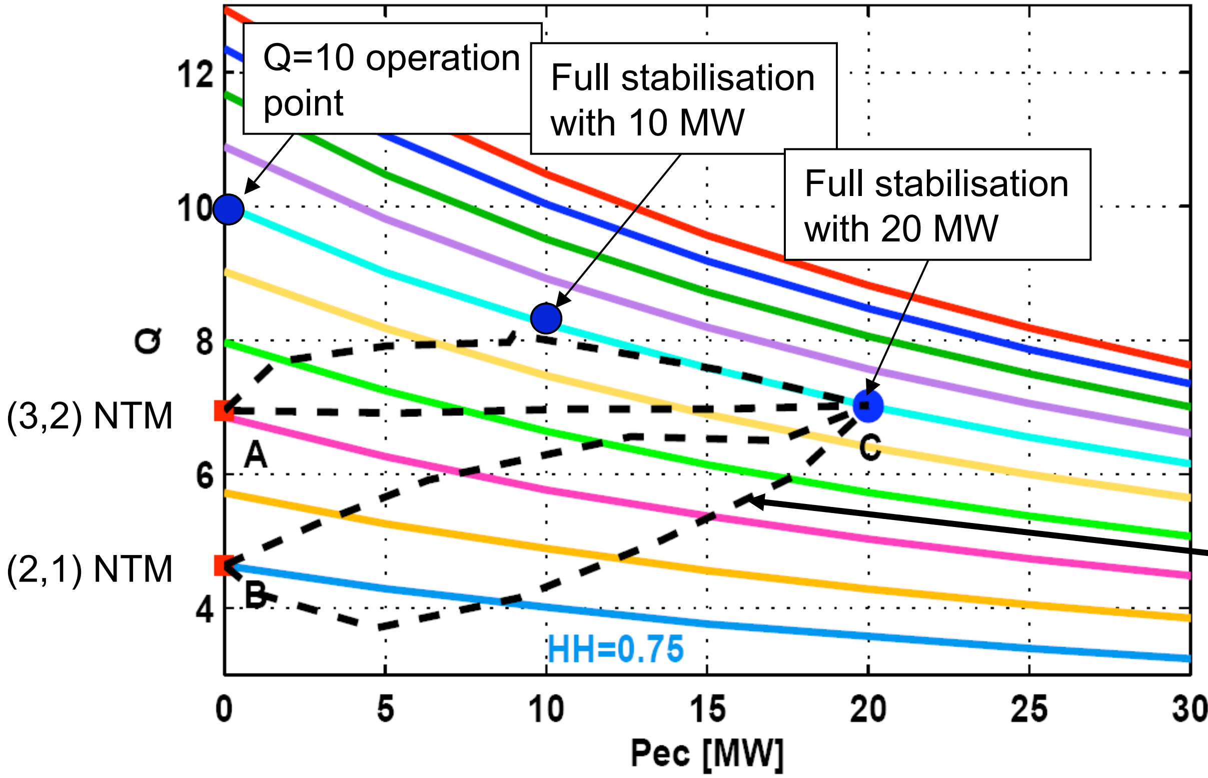
…by moving plasma major radius rigidly

3/2 NTM suppression sustained



Advantage of early application of ECCD in JT60-U

ITER NTMs stabilisation goals



ITER burn curves with ECCD at q=3/2 (A) and q=2 (B)

(O. Sauter, H. Zohm, EPS 2005)

HH = 1.0

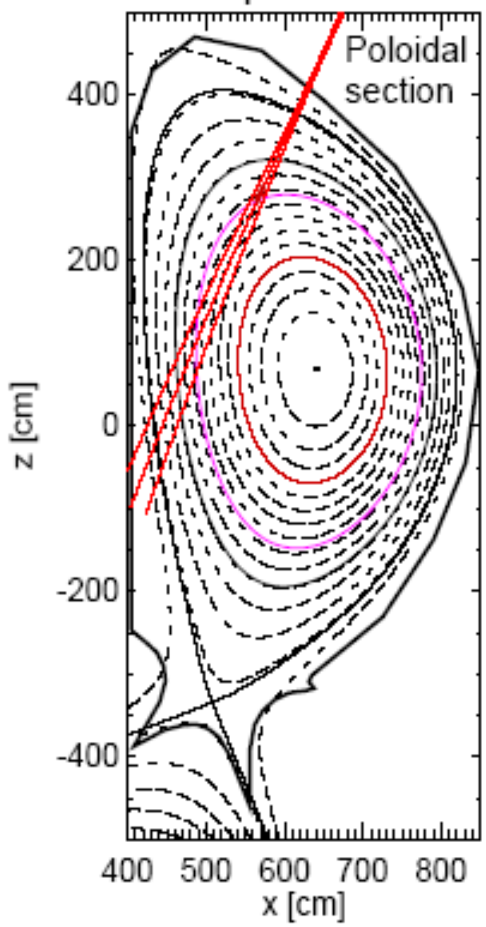
Sketch of paths with partial stabilisation

Impact on Q in case of continuous stabilisation (worst case):

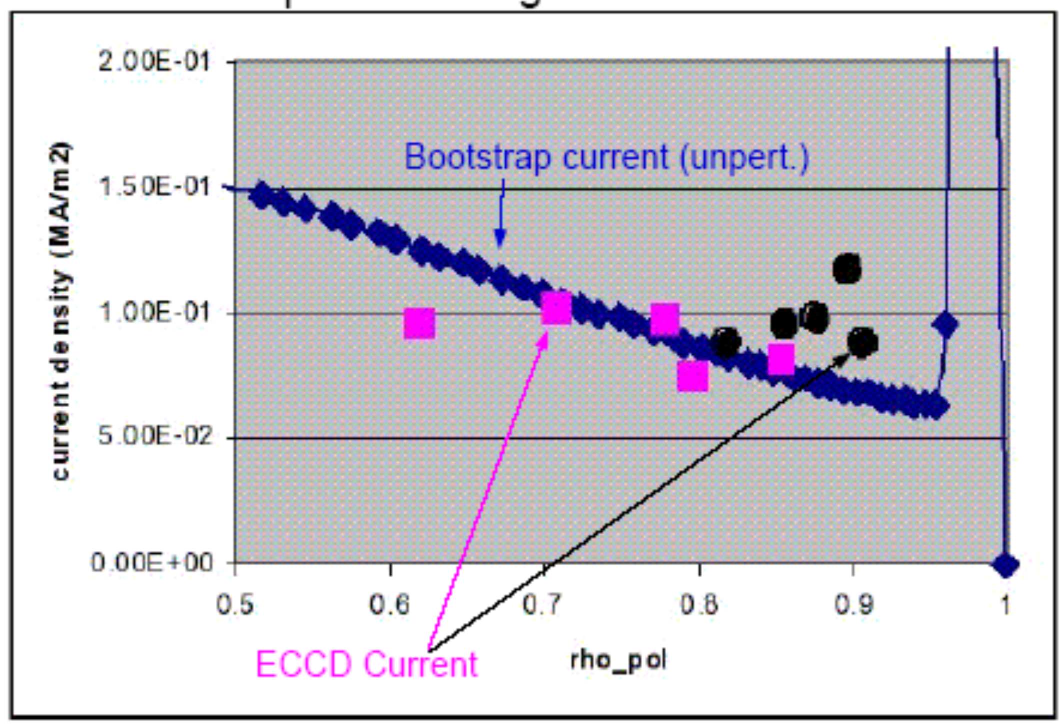
- Q drops from 10 to 5 for a (2,1) NTM and from 10 to 7 for (3,2) NTM
- with 20 MW needed for stabilisation, Q recovers to 7, with 10 MW to Q > 8
- note: if NTMs occur only occasionally, impact of ECCD on Q is small

Active NTM stabilisation in ITER

- Upper ECRH system for active stabilisation of (3,2) and (2,1) islands under development
- Current deposition calculated by means of the TORBEAM code [Poli et al., CPC 1999]

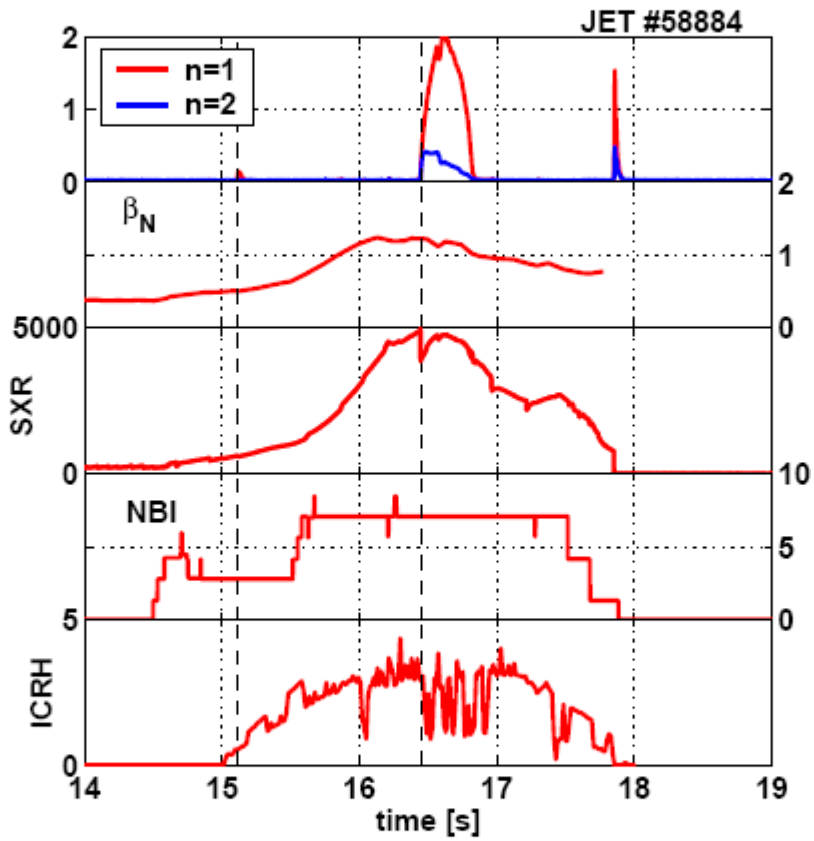


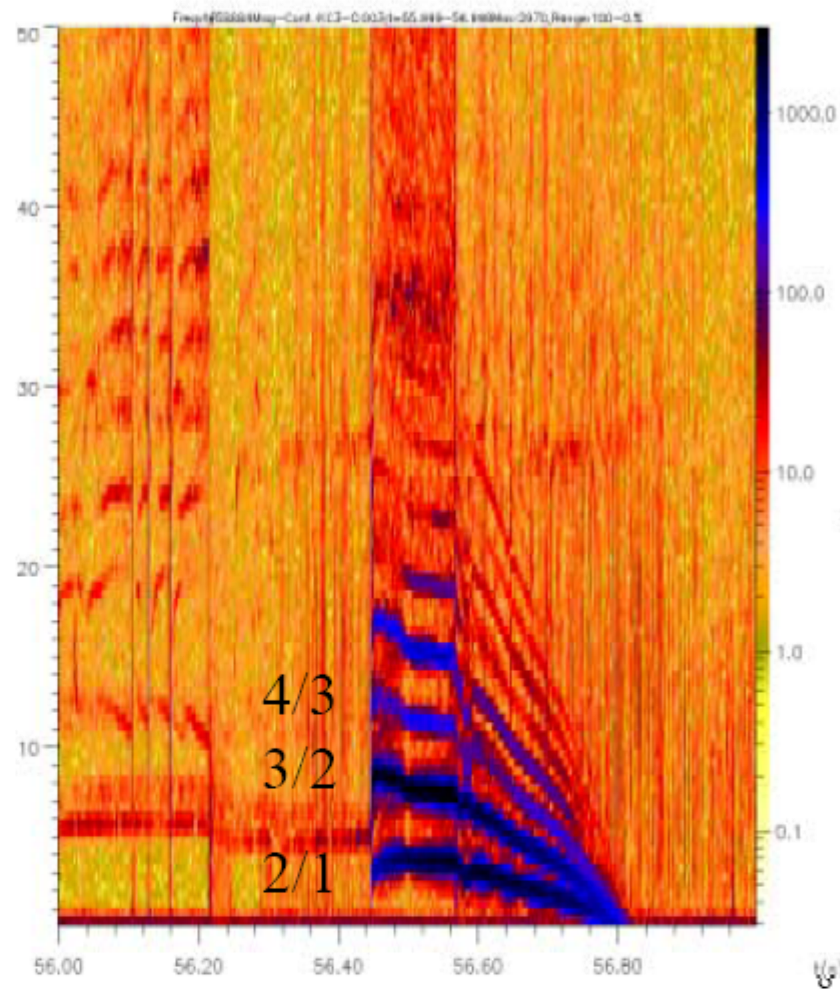
- Driven current smaller than the missing bootstrap current for the present design



[Zohm, Poli et al., EC13 (2004)]

Importance of trigger mechanism (1)



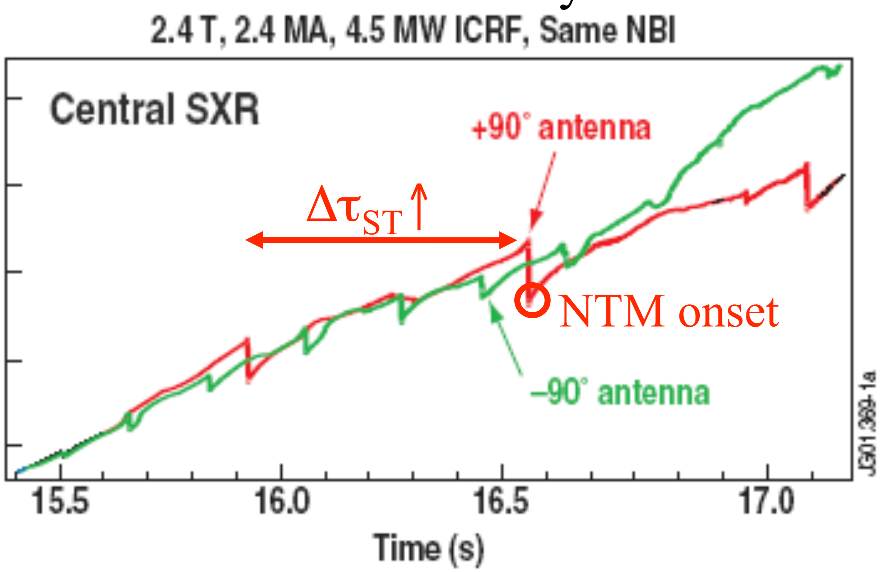


At sawtooth crash, many modes can be triggered

Importance of trigger mechanism (2)

Controlling sawteeth changes significantly β_{onset}

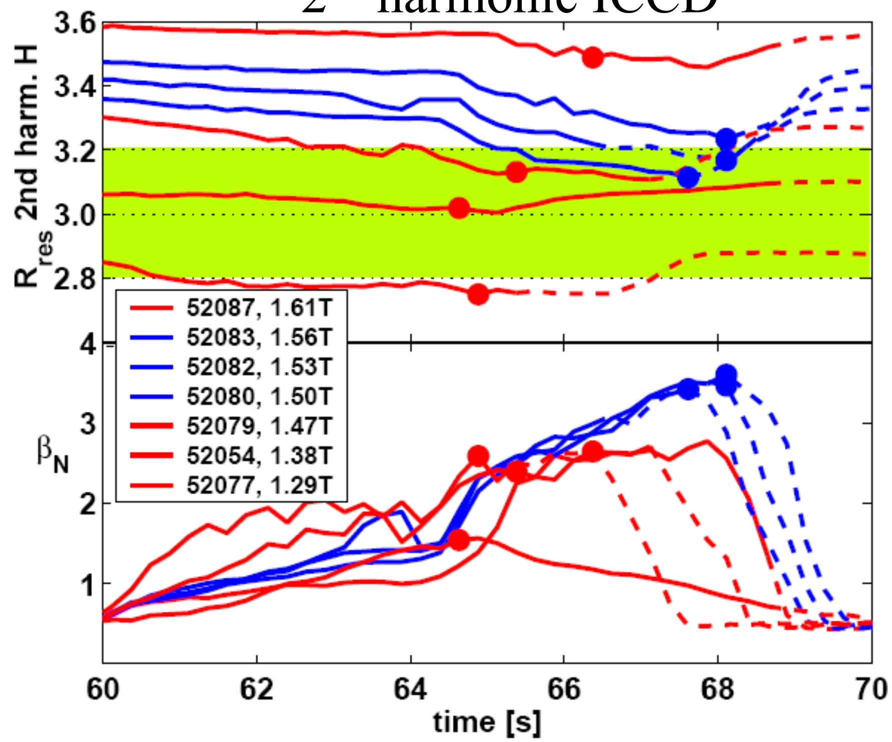
1st harmonic minority ICRH



+90: phase: $\beta_{N,onset}\approx 1$

-90: No NTM with β_N up to 2

2nd harmonic ICCD



Sauter et al, PRL 2002

EFDA JET

Can plasma flows help in the avoidance or control of NTMs?

How can flows affect NTMs?

- Flows can influence both outer layer and inner layer dynamics for resistive modes.

- They can also bring about changes in linear coupling mechanisms such as toroidal coupling between harmonics.

- Past nonlinear studies – mainly numerical – and often limited to simple situations (e.g. poloidal flows, non-self consistent) reveal interesting effects like oscillating islands, distortion in eigenfunctions etc.

- Also some recent analytic work on the effect of flow on the threshold and dynamical properties of magnetic islands which are relevant to NTMs

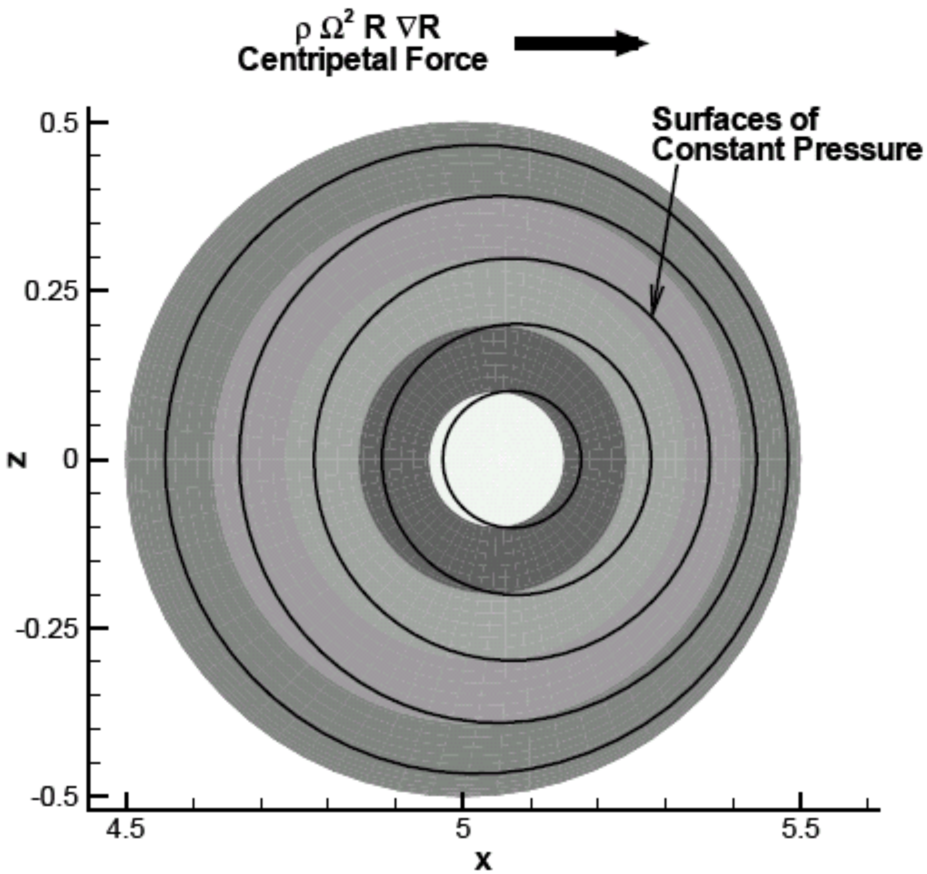
Refs: Chen &Morrison, '92, 94; Bondeson & Persson, '86,'88,'89; M.Chu,'98 Dewar & Persson, '93; Pletzer & Dewar, '90,'91,'94;Smolyakov '93,'95

- **Some recent experimental observations**

Main points of investigation

- Effects arising from equilibrium modifications
- Influence on toroidal coupling
- Influence on inner layer physics
- Changes in outer layer dynamics
- Nonlinear changes – saturation levels etc.

Equilibrium with toroidal flow

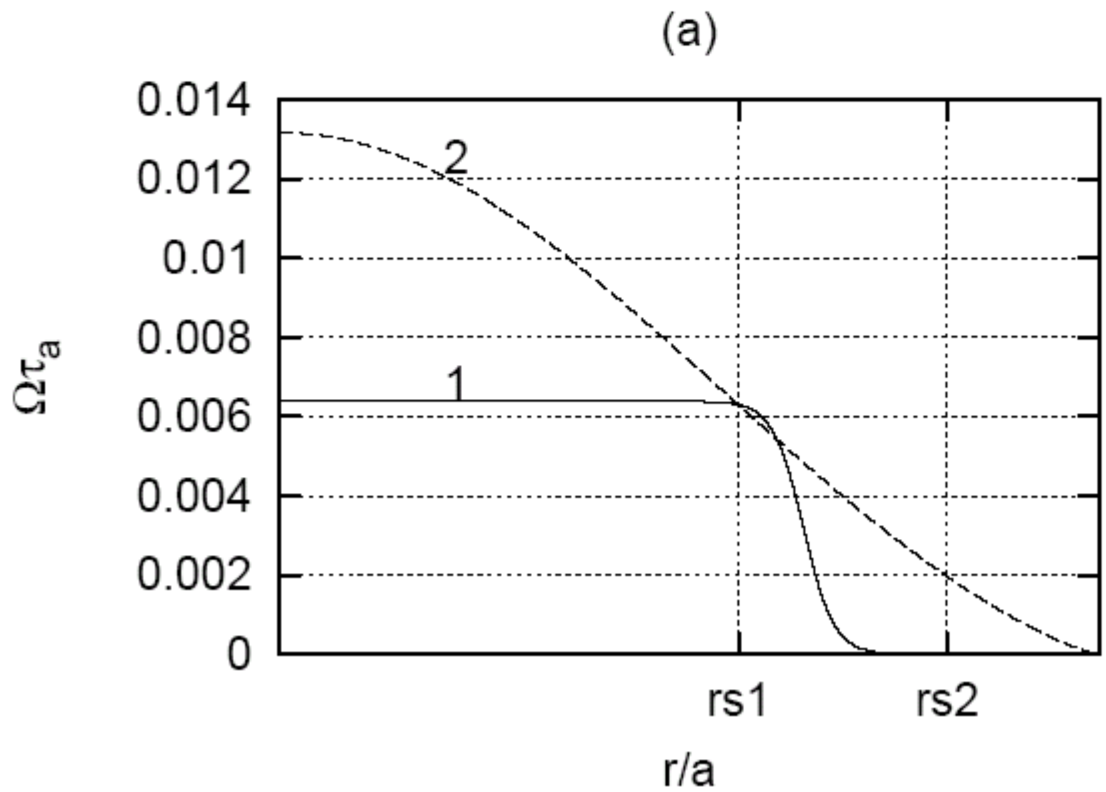


Constant pressure Surfaces shifted from Constant flux surfaces

$$p_0 = p_{nf}(\psi_0) \exp\left(\frac{\Gamma}{2} M_s^2(\psi_0)(\hat{R}^2 - \hat{R}^2_{axis})\right)$$

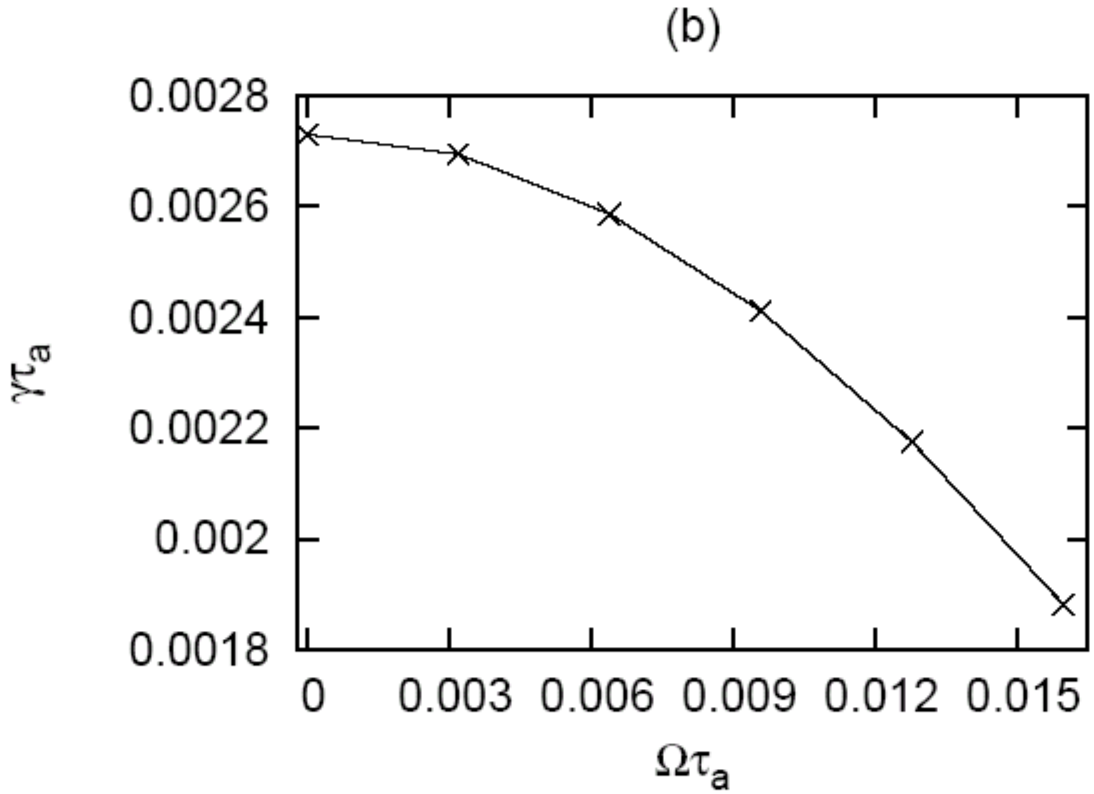
Maschke & Perrin, Plasma Phys. 22 (1980) 579

Toroidal flow profiles



1- differential flow
2- sheared flow

Reduction of (2,1) resistive TM growth with differential flow



- **stabilizing effect due to equilibrium changes**
  e.g. enhancement of pressure-curvature contribution
- **stabilizing effect due to flow induced de-coupling of rational surfaces**

- **Slab or cylinder**

$$\Delta'\Psi_s = -i\left(\omega - \Omega_s\right)\tau_L\Psi_s \; ; \quad \Omega_s = \vec{k}\cdot\vec{V}_0$$

$$\gamma = \frac{\Delta'}{\tau_L}$$

$$\Omega = \Omega_s$$

- **Toroidal geometry**

$$\Psi_{\mathbf{large}} = \Delta'\Psi$$

outer response - Δ' matrix

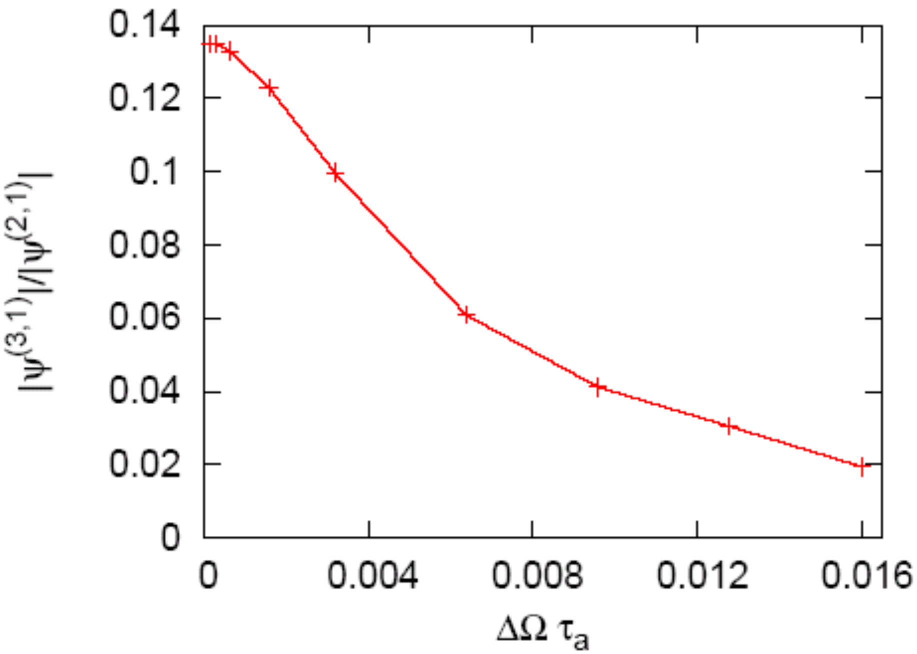
$$\Delta(\omega) = -i\left(\omega - \Omega_j\right)\tau_{Lj}\delta_{ij}$$

inner response

$$\det\begin{bmatrix} \Delta'_{11} - \Delta_{11}(\omega) & \Delta'_{12} \\ \Delta'_{21} & \Delta'_{22} - \Delta_{22}(\omega) \end{bmatrix} = 0.$$

Quadratic equation

Reduced reconection at the (3,1) surface



- **In the presence of finite flow shear the stabilization effect is smaller**

- This can be understood and explained quantitatively on the basis of linear slab theory analysis **(Chen & Morrison, PF B 2 (1990) 495)**

$$\gamma \sim \alpha^{2/5} \Delta'^{4/5} S^{-3/5} \hat{\gamma}$$

$$\hat{\gamma} = \text{flow correction} \geq 1$$

Small flow shear destabilizes the resistive mode through changes in the inner layer dynamics

Recent Experimental Observations

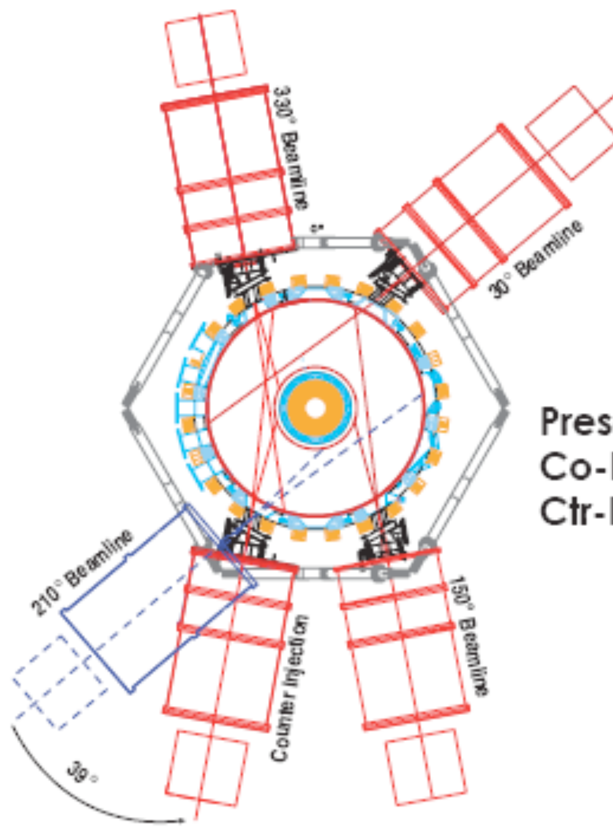
DIII-D Experiments

- **Near-toroidal beams inject energy and momentum**
  - ★ net torque varied by ratio of co to counter beams

- **Changes in tearing mode saturated amplitude observed**

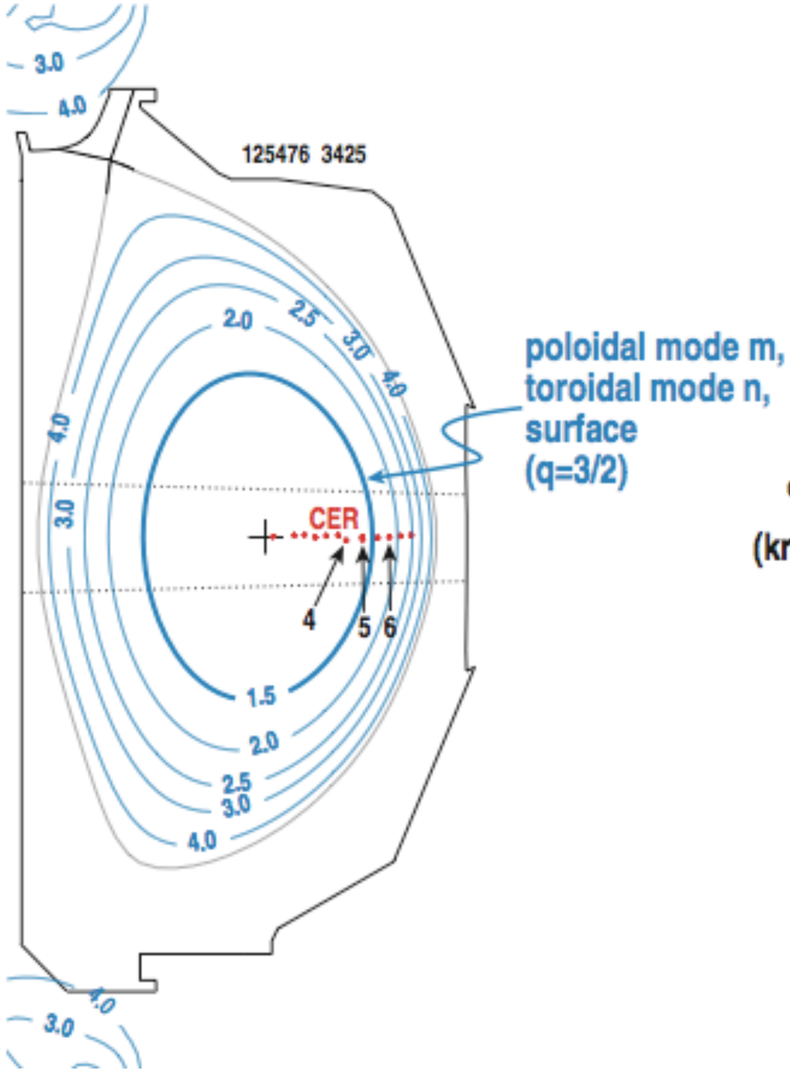
•hybrid scenario
•sawteething, ELMy H-mode

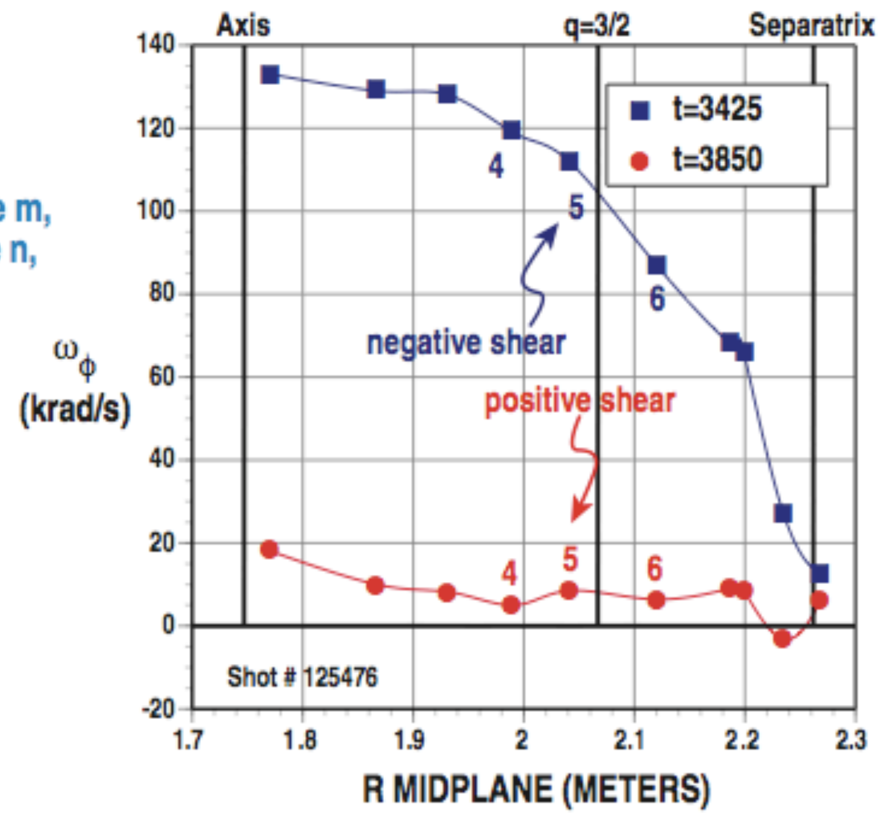
Plan View of DIII–D Tokamak



Present capability:
Co-NBI 12.5 MW
Ctr-NBI 5 MW

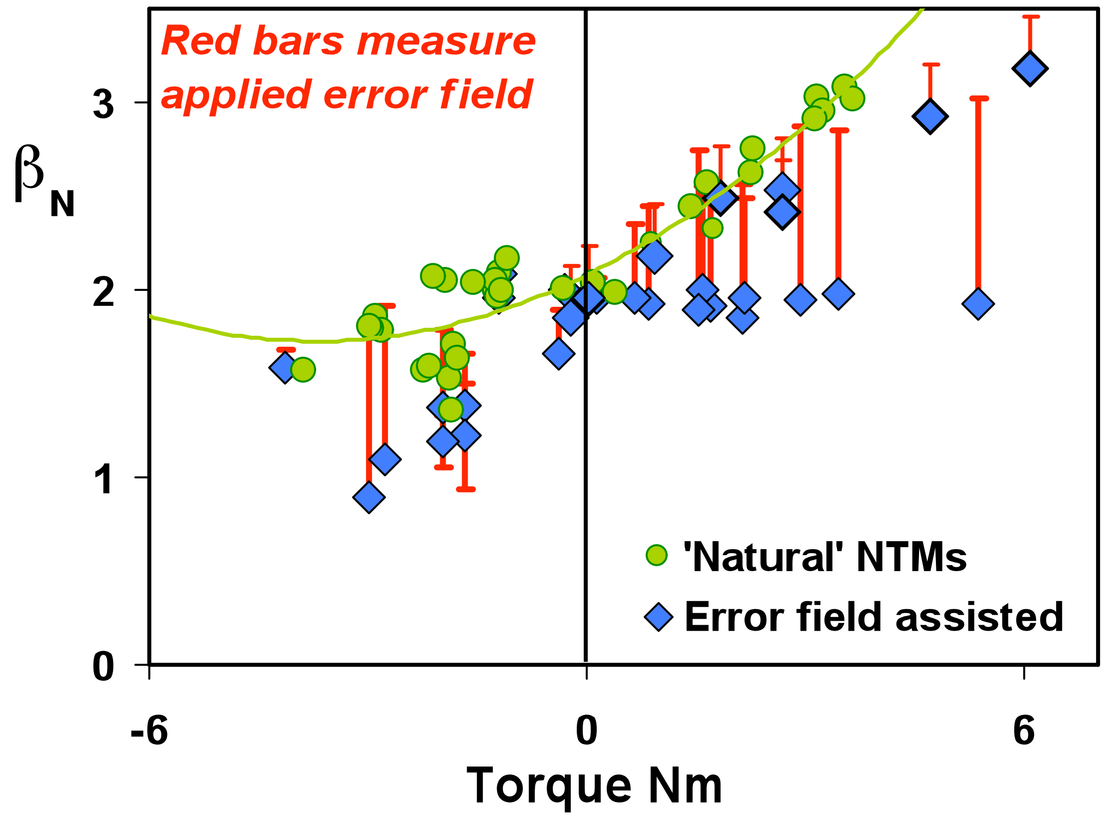
Plasma Rotation Measured by Charge Exchange Recombination of CVI Line





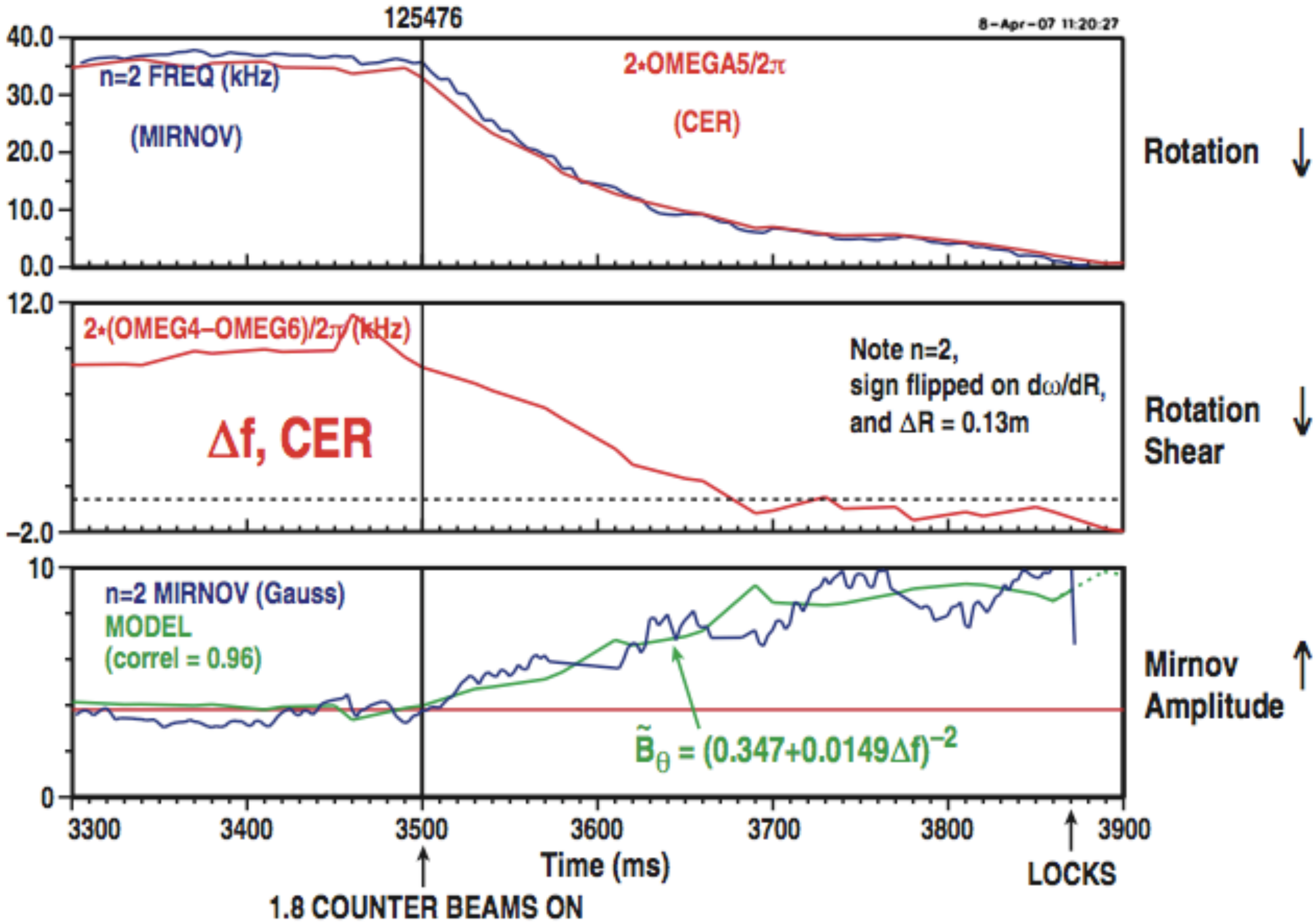
DIII D

DIII-D



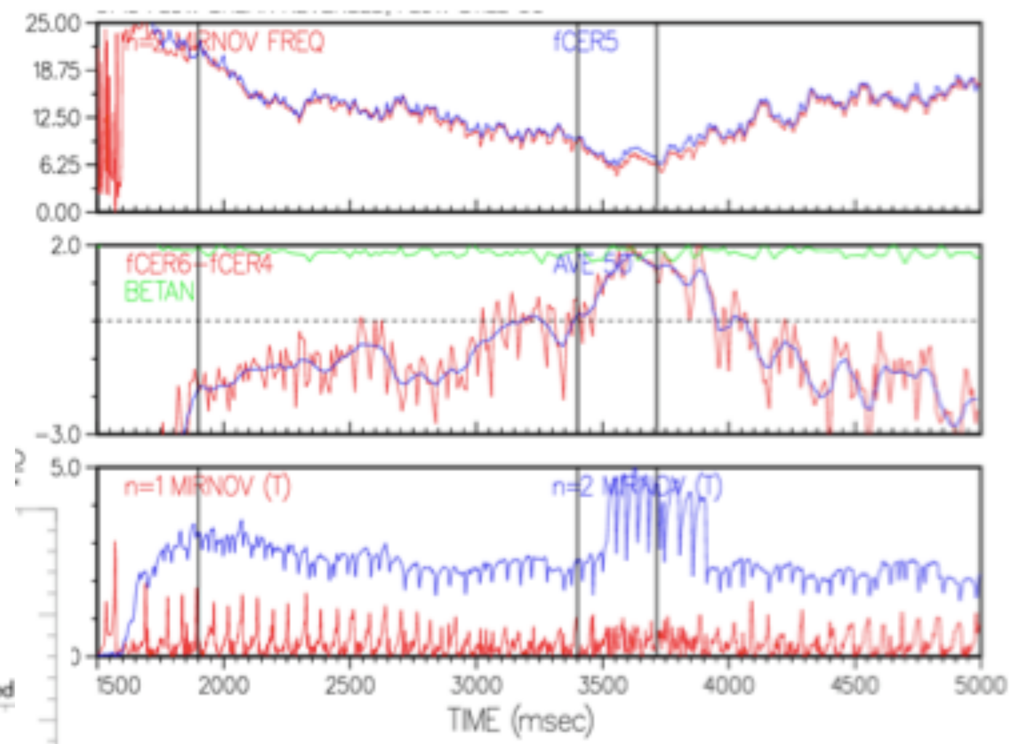
NTM onset has stronger drive (lower β_N) with lower rotation

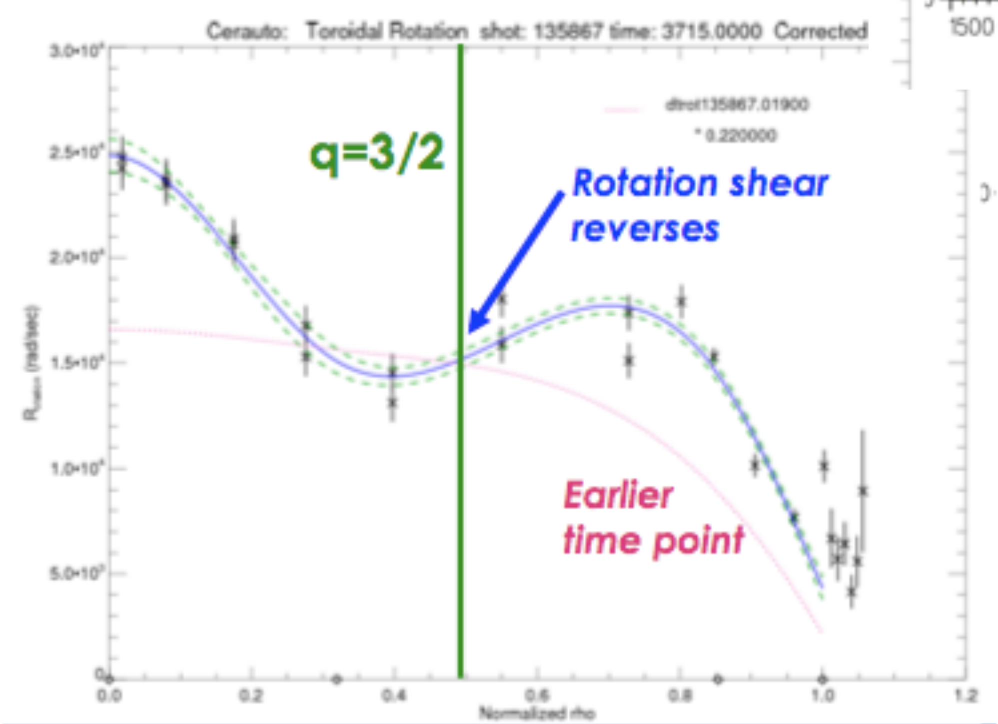
m/n=3/2 Hybrid Scenario NTM Bigger with Less Flow Shear



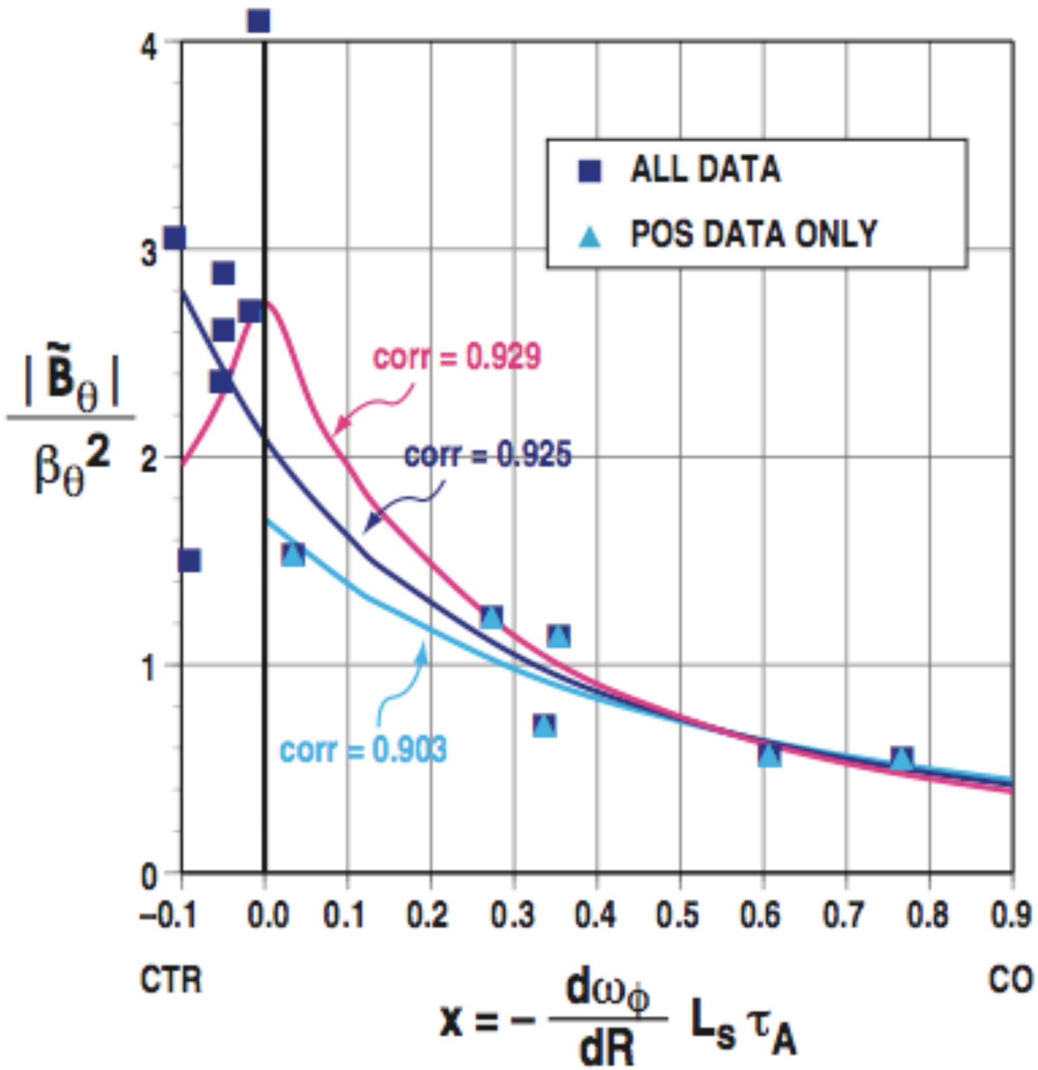
DIII-D

Rotation shear appears to play a crucial role on the dynamics of 3/2 NTMs. Sign of shear?





Reduction of 3/2 island size with increasing flow shear in Sawtoothing H mode discharges (DIII-D)



Experimental exploration of Rotation Effects on NTMs

- Similar observations have been made on other tokamaks e.g. JET, AUG, NSTX

- Joint experiments involving a number of machines and analysis involving multi-machine data currently underway as part of ITPA MHD Stability Topical Group initiative

- Story so far……

    - definite evidence of shear flow effect on NTM onset and saturation
    - some subtle differences between 2/1 and 3/2 behavior
    - dependence on sign of shear still an unresolved issue
    - Underlying mechanism?
        - inner layer / outer layer modification
        - linear/nonlinear
        - poloidal/toroidal

- Good theoretical understanding is lacking

Flow effects on the inner layer dynamics

- Two fluid model
- Flow terms are additional inertial contributions and modify the the polarization current term

The generalized Ohm's law

$$\underbrace{\mathbf{E}+\mathbf{v}\wedge\mathbf{B}}_{ideal\ MHD} = \underbrace{\eta\mathbf{j}}_{resistive\ MHD} + \underbrace{\frac{1}{\epsilon_0\omega_{pe}^2(1+\nu)}[\frac{\partial\mathbf{j}}{\partial t}+\nabla\ldots]}_{electron\ inertia} + \underbrace{\sum\frac{q_\alpha}{m_\alpha}(\nabla p_\alpha+\nabla\cdot\Pi_\alpha)}_{closures},$$

Modified Rutherford Equation for NTMs

Pressure/curvature

Neoclassical current

$$
\begin{aligned}
0.41\frac{\partial W}{\partial t} \quad = \quad & D_R^{neo}\left[\frac{\Delta_c'}{4} - \frac{19.5}{W}\frac{\epsilon L_s^2}{B_0^2}\frac{\partial p(0)}{\partial \psi} + 0.58\frac{\sqrt{\epsilon}\beta_\theta\frac{L_q}{L_p}}{W}\frac{W^2}{W^2+W_\chi^2}\right. \\
& \left. + \frac{L_s^2}{k_\theta^2 v_A^2}\left(2.3\frac{(\omega-\omega_E)(\omega-\omega_E-\omega_*)}{W^3} + 0.24\frac{{\omega_E'}^2}{W}\right) - 0.77\frac{L_s}{k_\theta v_A}\frac{\bar{v}_{\|0}}{v_A}\frac{\omega_E'}{W}\right]
\end{aligned}
$$

differential flow

flow shear

polarization current

$$W_{sat} \sim \frac{\beta_\theta}{(-\Delta')} \frac{L_q}{L_p}$$

Experimental evidence suggests that β_θ and $\frac{L_q}{L_p}$ do not change significantly with changing flow

So something could be happening to Δ'

What is the dependence of Δ' on flow shear?

Heuristic Model

- rotation shear provides additional drive to alter field line pitch
- can increase or decrease field line bending energy and thereby change Δ'

$$\Delta' r_s = C_1 + C_2 \left(-\frac{d\omega_\phi}{dR} L_s \tau_A \right)$$

Simplest empirical form

Can one see this scaling from theoretical models ?

- RMHD code
- Newcomb eqn. with flow

Code NEAR

- NEAR – fully nonlinear toroidal code that solves a set of RMHD eqns. and contains neoclassical viscous terms as well as toroidal flow

- Has been benchmarked to reproduce linear (classical) tearing mode dynamics as well as nonlinear saturated behaviour

- It has also reproduced well the dynamics of NTMs – e.g. threshold dynamics, scaling with β_p, island saturation etc.

(**D. Chandra, A. Sen, P. Kaw, M.P. Bora and S. Kruger, Nuc. Fus. 45 (2005) 524**)

- Have examined the scaling of Δ' with toroidal flow shear for classical tearing modes

Model Equations (GRMHD)

$$\frac{\partial \Psi}{\partial t} - (\boldsymbol{b}_0 + \boldsymbol{b}_1) \cdot \nabla \phi_1 - \boldsymbol{b}_1 \cdot \nabla \phi_0 = \eta \tilde{J}_{||} - \frac{1}{ne} \boldsymbol{b}_0 \cdot \nabla \cdot \boldsymbol{\Pi}_e$$

bootstrap current

$$\nabla \cdot \left(\frac{\rho}{B_0} \frac{d}{dt} \frac{\nabla \phi_1}{B_0} \right) + (\boldsymbol{V}_1 \cdot \nabla) \left(\nabla \cdot \left(\frac{\rho}{B_0} \frac{\nabla \phi_0}{B_0} \right) \right) = (\boldsymbol{B}_0 \cdot \nabla) \frac{\tilde{J}_{||}}{B_0} + (\boldsymbol{B}_1 \cdot \nabla) \frac{J_{T||}}{B_0} + \nabla \cdot \frac{\boldsymbol{B}_0 \times \nabla p_1}{B_0^2} + \nabla \cdot \frac{\boldsymbol{B}_0}{B_0^2} \times \nabla \cdot \boldsymbol{\Pi}$$

GGJ

$$\frac{dp_1}{dt} + (\boldsymbol{V}_1 \cdot \nabla) p_0 + \Gamma p_T \nabla \cdot \boldsymbol{V}_1 = (\Gamma - 1) \left[\eta J_{T||}^2 - \boldsymbol{\Pi} : \nabla \boldsymbol{V} + \boldsymbol{\Pi_e} : \nabla \frac{\boldsymbol{J}}{ne} - \nabla \cdot \boldsymbol{q} \right]$$

heat flow

$$\rho \frac{d\widetilde{V_{||}}}{dt} + (\boldsymbol{V}_1 \cdot \nabla) V_{||_0} = -\boldsymbol{b}_0 \cdot \nabla p_1 - \boldsymbol{b}_1 \cdot \nabla p_T - \boldsymbol{b}_0 \cdot \nabla \cdot \boldsymbol{\Pi}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{V} \cdot \nabla$$

$$\boldsymbol{V} = \Omega(\psi) R^2 \boldsymbol{\nabla}\zeta + \boldsymbol{V}_1 = \frac{\boldsymbol{B}_0 \times \nabla\Phi_0}{B_0^2} + V_{0||}\boldsymbol{b}_0 + \frac{\boldsymbol{B}_0 \times \nabla\Phi_1}{B_0^2} + \tilde{V}_{||}\boldsymbol{b}_T$$

Equilibrium flow

- **Neoclassical closure**

$$\vec{\nabla} \cdot \Pi_s = \rho_s \mu_s \left\langle B^2 \right\rangle \frac{\vec{V}_s \cdot \vec{\nabla}\Theta}{\left(\vec{B} \cdot \vec{\nabla}\Theta\right)^2} \vec{\nabla}\Theta,$$

- appropriate for long mean free path limit
- reproduces poloidal flow damping
- gives appropriate perturbed bootstrap current

Numerical simulation

- **GRMHD** eqns solved using code *NEAR* – toroidal initial value code – Fourier decomposition in the poloidal and toroidal directions and central finite differencing in the flux coordinate direction.

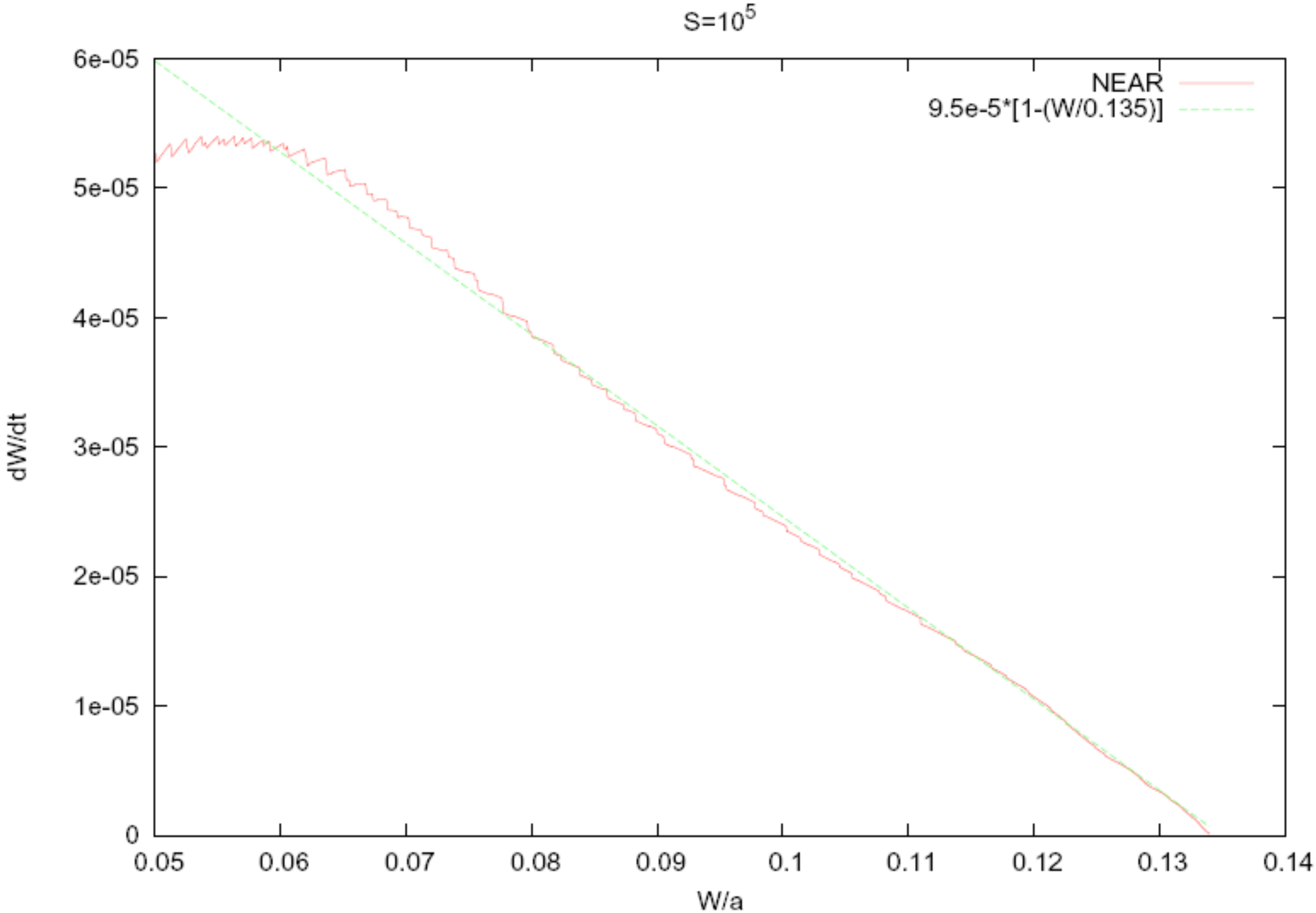
- Equilibrium generated from another independent code **TOQ**

- Typical runs are made at **$S \sim 10^5$, low β, sub-Alfvenic flows**

- Linear benchmarking done for classical resistive modes

- For NTMs threshold, island saturation etc. benchmarked in the absence of flows.

- Present study restricted to **sheared toroidal flows**


S=10^5
6e-05
5e-05
4e-05
3e-05
2e-05
1e-05
0
dW/dt
0.05
0.06
0.07
0.08
0.09
0.1
0.11
0.12
0.13
0.14
W/a
NEAR
9.5e-5*[1-(W/0.135)]

Determination of Δ'

- Linear growth rate :

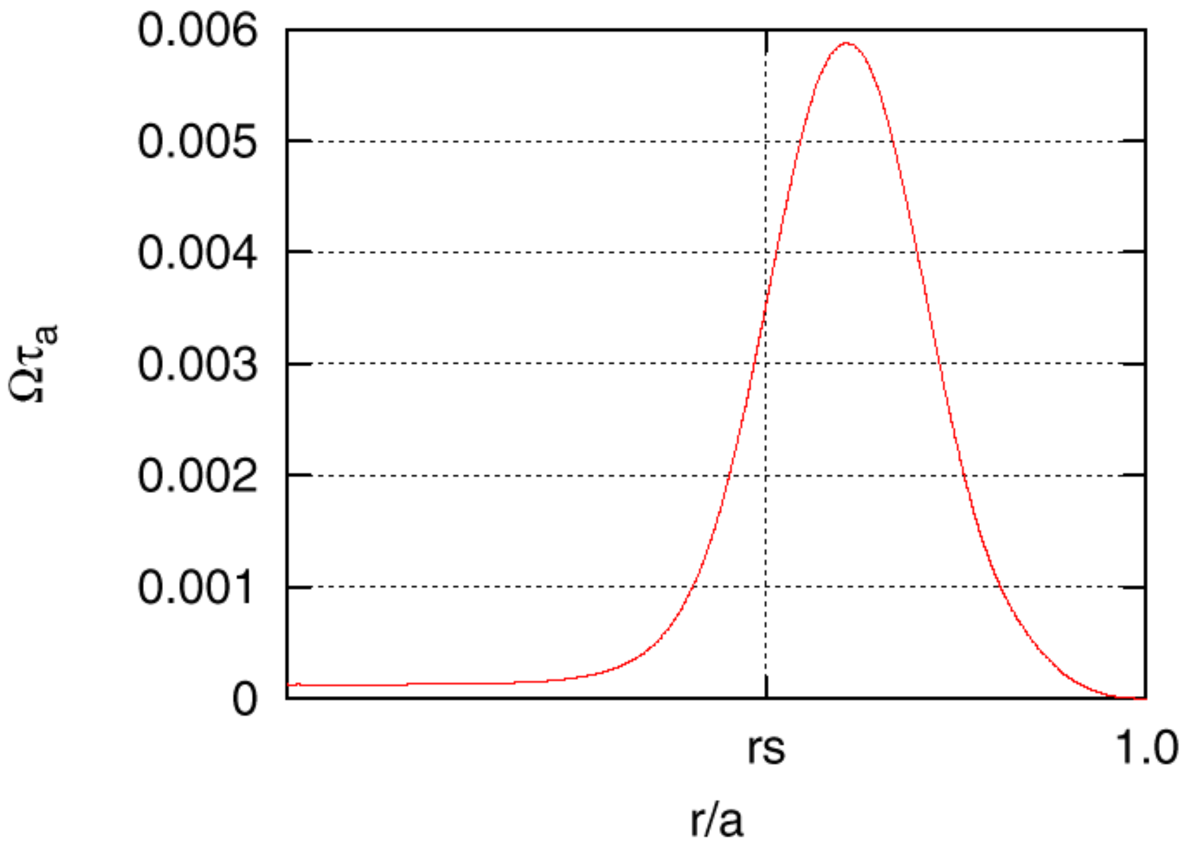
$$\gamma = C\,(\Delta')^{4/5}\,S^{-3/5}$$

- Nonlinear growth close to saturation

$$\frac{dW}{dt} = \Delta'\left(1 - \frac{W}{W_{sat}}\right)$$

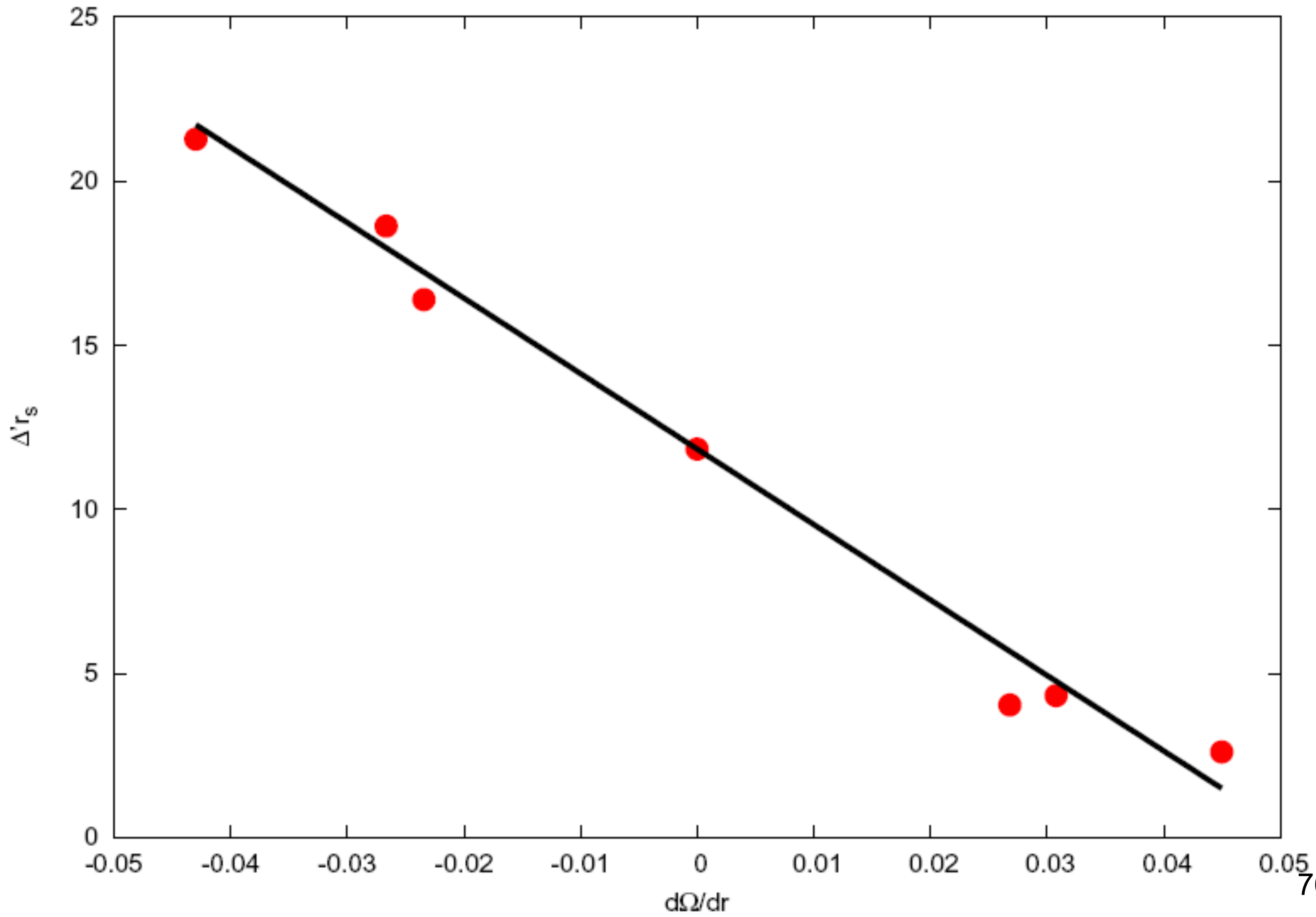
- Cross check linear and nonlinear results without flow and make runs with flow

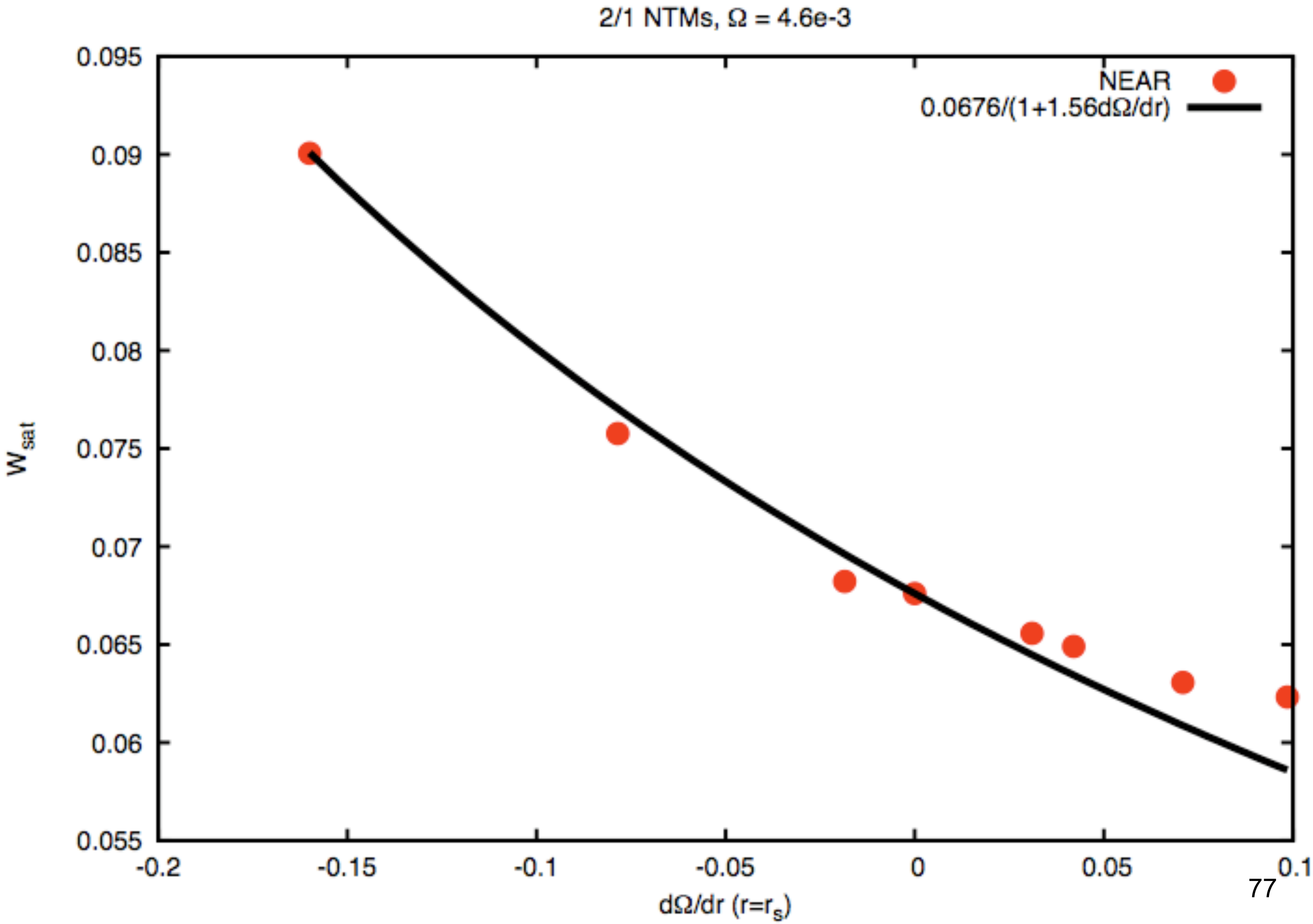
Profile with positive flow shear at (2,1) surface



- Looked at single helicity mode dynamics

Results from NEAR





2/1 NTMs, Ω = 4.6e-3
NEAR
0.0676/(1+1.56dΩ/dr)
0.095
0.09
0.085
0.08
0.075
0.07
0.065
0.06
0.055
W_{sat}
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
dΩ/dr (r=r_s)

Newcomb Equation with sheared flow:

$$H\frac{d^2\psi}{dr^2}+\left(\frac{dH}{dr}+h_f\right)\frac{d\psi}{dr}-\left[\frac{g}{F^2}+\frac{g_f}{F^2}+\frac{1}{F}\frac{d}{dr}\left(H\frac{dF}{dr}\right)\right]\psi=0$$

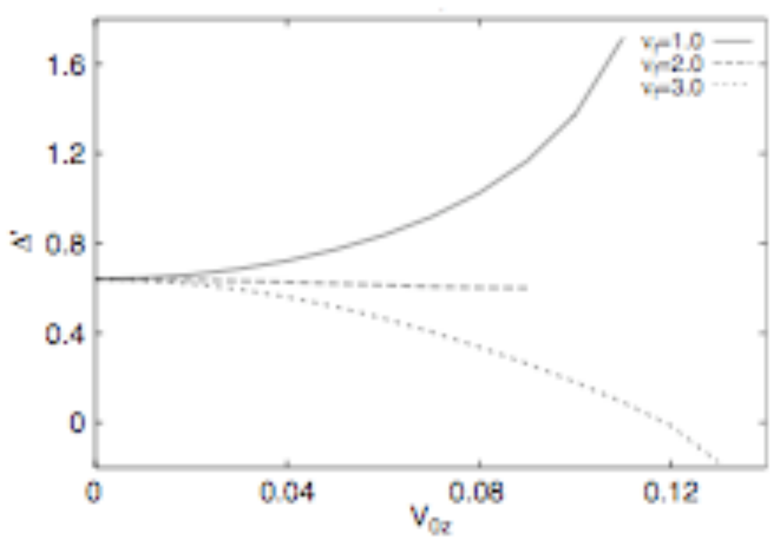
h_f and **g_f** are additional contributions due to flow

- **Limit: h_f , g_f → 0,** Furth, Rutherford, Selberg equation
  *[Phys. Fluids **16**, 1054 (1973)]*

- **Limit: slab geometry, (1/r) → 0, d/dr → d/dx, m/r → k_y**
  Chen-Morrison Equation *[Phys. Fluids B **2**, 495 (1990)]*

$$\Delta' \;=\; -\frac{1}{r_s\psi_s^2}\int_0^a\left[\left(\frac{d\psi}{dr}\right)^2+\left\{\frac{g}{HF^2}+\frac{1}{HF}\frac{d}{dr}\left(H\frac{dF}{dr}\right)-\frac{2m^2k_z^2}{(k_z^2r^2+m^2)^2}\right.\right.$$
$$\left.\left.+\frac{g_f}{HF^2}+\frac{1}{2r}\frac{d}{dr}\left(\frac{rh_f}{H}\right)\right\}\psi^2\right]rdr$$

- The value of Δ' quite sensitive to the magnetic and flow profiles



- Quantitative comparisons with NEAR results are presently in progress

- Necessary to carry out better numerical investigations e.g. using PEST3 or other codes and from Newcomb's equation

- Need analytic modeling for better understanding of the underlying physics

- On going activity within the ITPA MHD Topical Group including effect of flow on the sawtooth instability

Outstanding Theoretical and Experimental Issues for NTMs

- **Island width threshold**

  - perpendicular heat transport - local model - improvements necessary - active ongoing theoretical effort

  - neoclassical/ion polarization effects - several open theoretical questions (role of drift waves, ion viscosity effects at high temp, the exact value of the mode frequency, role of energetic ions etc.) - experimental determination also a challenge.

•Seed Island formation

- `standard' NTM initiated by outside MHD event - proper modeling necessary

- 'seedless' NTMs have been seen on TFTR/MAST

    - coupling to an ideal perturbed mode

    - Δ' > 0 modes nonlinearly saturating at small levels?

    - Small scale islands modulated by ion population?

    - turbulence induced trigger

- **Local Current Drive stabilization**
  - works well when island O point is hit - optimization methods being worked out.

- **Non-resonant Helical perturbation**
  - works well experimentally but mechanism not well understood theoretically
  - slows down rotation - affects other modes e.g. resistive wall mode

- **Interaction of fast particles with NTMs** – open problem

- **Plasma Rotation Effects on NTM** – open problem

New NTM regime – Frequently Interrupted Regime

- Happens at higher $\beta_N > 2.3$
- Growth of the NTM is often interrupted by drops in amplitude
- Observed for (3,2) modes in AUG and JET
- Confinement degradation is markedly reduced – so a benign regime
- Possible mechanism – nonlinear coupling between (3,2) NTM, (1,1) and (4,3) mode.

Concluding Remarks

- NTMs are large size magnetic islands driven by neoclassical effects
- Basic physics fairly well understood - modified Rutherford eqn.
- Can have a major impact on tokamak performance by **limiting β**
- Experimentally widely observed in several tokamaks
- **ECCD method of stabilization works** well and is understood
- Still many experimental features (seed island, FJs, non-resonant stabilization etc.) are not well understood.
- **Active area of research offering opportunities for theoretical and experimental insight into reconnection and MHD control issues.**